Chapter 14

Fluid Mechanics

CHAPTER OUTLINE

▲ *These hot-air balloons float because they are filled with air at high temperature and are surrounded by denser air at a lower temperature. In this chapter, we will explore the buoyant force that supports these balloons and other floating objects. (Richard Megna/Fundamental Photographs)*

Matter is normally classified as being in one of three states: solid, liquid, or gas. From everyday experience, we know that a solid has a definite volume and shape. A brick maintains its familiar shape and size day in and day out. We also know that a liquid has a definite volume but no definite shape. Finally, we know that an unconfined gas has neither a definite volume nor a definite shape. These descriptions help us picture the states of matter, but they are somewhat artificial. For example, asphalt and plastics are normally considered solids, but over long periods of time they tend to flow like liquids. Likewise, most substances can be a solid, a liquid, or a gas (or a combination of any of these), depending on the temperature and pressure. In general, the time it takes a particular substance to change its shape in response to an external force determines whether we treat the substance as a solid, a liquid, or a gas.

A **fluid** is a collection of molecules that are randomly arranged and held together by weak cohesive forces and by forces exerted by the walls of a container. Both liquids and gases are fluids.

In our treatment of the mechanics of fluids, we do not need to learn any new physical principles to explain such effects as the buoyant force acting on a submerged object and the dynamic lift acting on an airplane wing. First, we consider the mechanics of a fluid at rest—that is, *fluid statics.* We then treat the mechanics of fluids in motion—that is, *fluid dynamics.* We can describe a fluid in motion by using a model that is based upon certain simplifying assumptions.

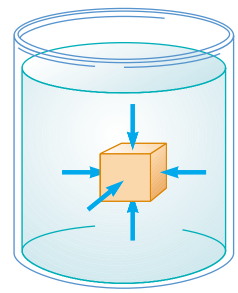


Figure 14.1 At any point on the surface of a submerged object, the force exerted by the fluid is perpendicular to the surface of the object. The force exerted by the fluid on the walls of the container is perpendicular to the walls at all points.

14.1 Pressure

Fluids do not sustain shearing stresses or tensile stresses; thus, the only stress that can be exerted on an object submerged in a static fluid is one that tends to compress the object from all sides. In other words, the force exerted by a static fluid on an object is always perpendicular to the surfaces of the object, as shown in Figure 14.1.

The pressure in a fluid can be measured with the device pictured in Figure 14.2. The device consists of an evacuated cylinder that encloses a light piston connected to a spring. As the device is submerged in a fluid, the fluid presses on the top of the piston and compresses the spring until the inward force exerted by the fluid is balanced by the outward force exerted by the spring. The fluid pressure can be measured directly if the spring is calibrated in advance. If F is the magnitude of the force exerted on the piston and A is the surface area of the piston, then the **pressure** P of the fluid at the level to which the device has been submerged is defined as the ratio F/A:

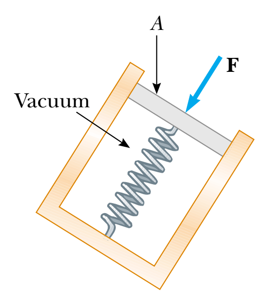


Figure 14.2 A simple device for measuring the pressure exerted by a fluid.

Definition of pressure

$$P \equiv \frac{F}{A} \tag{14.1}$$

Note that pressure is a scalar quantity because it is proportional to the magnitude of the force on the piston.

If the pressure varies over an area, we can evaluate the infinitesimal force dF on an infinitesimal surface element of area dA as

Earl Young/Getty Images

Snowshoes keep you from sinking into soft snow because they spread the downward force you exert on the snow over a large area, reducing the pressure on the snow surface.

PITFALL PREVENTION

14.1 Force and Pressure

Equations 14.1 and 14.2 make a clear distinction between force and pressure. Another important distinction is that *force is a vector* and *pressure is a scalar.* There is no direction associated with pressure, but the direction of the force associated with the pressure is perpendicular to the surface of interest.

$$dF = P\,dA \tag{14.2}$$

where P is the pressure at the location of the area dA. The pressure exerted by a fluid varies with depth. Therefore, to calculate the total force exerted on a flat vertical wall of a container, we must integrate Equation 14.2 over the surface area of the wall.

Because pressure is force per unit area, it has units of newtons per square meter (N/m^2) in the SI system. Another name for the SI unit of pressure is **pascal** (Pa):

$$1\ \text{Pa} \equiv 1\ \text{N/m}^2 \tag{14.3}$$

Quick Quiz 14.1 Suppose you are standing directly behind someone who steps back and accidentally stomps on your foot with the heel of one shoe. Would you be better off if that person were (a) a large professional basketball player wearing sneakers (b) a petite woman wearing spike-heeled shoes?

Example 14.1 The Water Bed

The mattress of a water bed is 2.00 m long by 2.00 m wide and 30.0 cm deep.

(A) Find the weight of the water in the mattress.

Solution The density of fresh water is 1 000 kg/m^3 (see Table 14.1 on page 423), and the volume of the water filling the mattress is $V = (2.00\ \text{m})(2.00\ \text{m})(0.300\ \text{m}) = 1.20\ \text{m}^3$. Hence, using Equation 1.1, the mass of the water in the bed is

$$M = \rho V = (1\,000\ \text{kg/m}^3)(1.20\ \text{m}^3) = 1.20 \times 10^3\ \text{kg}$$

and its weight is

$$Mg = (1.20 \times 10^3\ \text{kg})(9.80\ \text{m/s}^2) = \boxed{1.18 \times 10^4\ \text{N}}$$

This is approximately 2 650 lb. (A regular bed weighs approximately 300 lb.) Because this load is so great, such a water bed is best placed in the basement or on a sturdy, well-supported floor.

(B) Find the pressure exerted by the water on the floor when the bed rests in its normal position. Assume that the entire lower surface of the bed makes contact with the floor.

Solution When the bed is in its normal position, the area in contact with the floor is 4.00 m^2; thus, from Equation 14.1, we find that

$$P = \frac{1.18 \times 10^4\ \text{N}}{4.00\ \text{m}^2} = \boxed{2.95 \times 10^3\ \text{Pa}}$$

What If? **What if the water bed is replaced by a 300-lb ordinary bed that is supported by four legs? Each leg has a circular cross section of radius 2.00 cm. What pressure does this bed exert on the floor?**

Answer The weight of the bed is distributed over four circular cross sections at the bottom of the legs. Thus, the pressure is

$$P = \frac{F}{A} = \frac{mg}{4(\pi r^2)} = \frac{300\ \text{lb}}{4\pi(0.0200\ \text{m})^2}\left(\frac{1\ \text{N}}{0.225\ \text{lb}}\right)$$
$$= 2.65 \times 10^5\ \text{Pa}$$

Note that this is almost 100 times larger than the pressure due to the water bed! This is because the weight of the ordinary bed, even though it is much less than the weight of the water bed, is applied over the very small area of the four legs. The high pressure on the floor at the feet of an ordinary bed could cause denting of wood floors or permanently crush carpet pile. In contrast, a water bed requires a sturdy floor to support the very large weight.

Table 14.1

Densities of Some Common Substances at Standard Temperature (0°C) and Pressure (Atmospheric)

Substance	ρ (kg/m^3)	Substance	ρ (kg/m^3)
Air	1.29	Ice	0.917×10^3
Aluminum	2.70×10^3	Iron	7.86×10^3
Benzene	0.879×10^3	Lead	11.3×10^3
Copper	8.92×10^3	Mercury	13.6×10^3
Ethyl alcohol	0.806×10^3	Oak	0.710×10^3
Fresh water	1.00×10^3	Oxygen gas	1.43
Glycerin	1.26×10^3	Pine	0.373×10^3
Gold	19.3×10^3	Platinum	21.4×10^3
Helium gas	1.79×10^{-1}	Seawater	1.03×10^3
Hydrogen gas	8.99×10^{-2}	Silver	10.5×10^3

14.2 Variation of Pressure with Depth

As divers well know, water pressure increases with depth. Likewise, atmospheric pressure decreases with increasing altitude; for this reason, aircraft flying at high altitudes must have pressurized cabins.

We now show how the pressure in a liquid increases with depth. As Equation 1.1 describes, the *density* of a substance is defined as its mass per unit volume; Table 14.1 lists the densities of various substances. These values vary slightly with temperature because the volume of a substance is temperature-dependent (as shown in Chapter 19). Under standard conditions (at 0°C and at atmospheric pressure) the densities of gases are about 1/1 000 the densities of solids and liquids. This difference in densities implies that the average molecular spacing in a gas under these conditions is about ten times greater than that in a solid or liquid.

Now consider a liquid of density ρ at rest as shown in Figure 14.3. We assume that ρ is uniform throughout the liquid; this means that the liquid is incompressible. Let us select a sample of the liquid contained within an imaginary cylinder of cross-sectional area A extending from depth d to depth $d + h$. The liquid external to our sample exerts forces at all points on the surface of the sample, perpendicular to the surface. The pressure exerted by the liquid on the bottom face of the sample is P, and the pressure on the top face is P_0. Therefore, the upward force exerted by the outside fluid on the bottom of the cylinder has a magnitude PA, and the downward force exerted on the top has a magnitude P_0A. The mass of liquid in the cylinder is $M = \rho V = \rho Ah$; therefore, the weight of the liquid in the cylinder is $Mg = \rho Ahg$. Because the cylinder is in equilibrium, the net force acting on it must be zero. Choosing upward to be the positive y direction, we see that

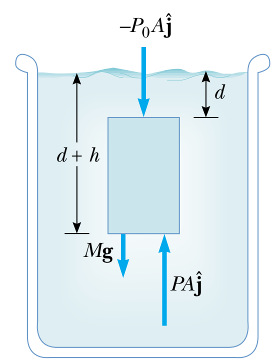


Figure 14.3 A parcel of fluid (darker region) in a larger volume of fluid is singled out. The net force exerted on the parcel of fluid must be zero because it is in equilibrium.

$$\sum \mathbf{F} = PA\hat{\mathbf{j}} - P_0A\hat{\mathbf{j}} - M\mathbf{g}\hat{\mathbf{j}} = 0$$

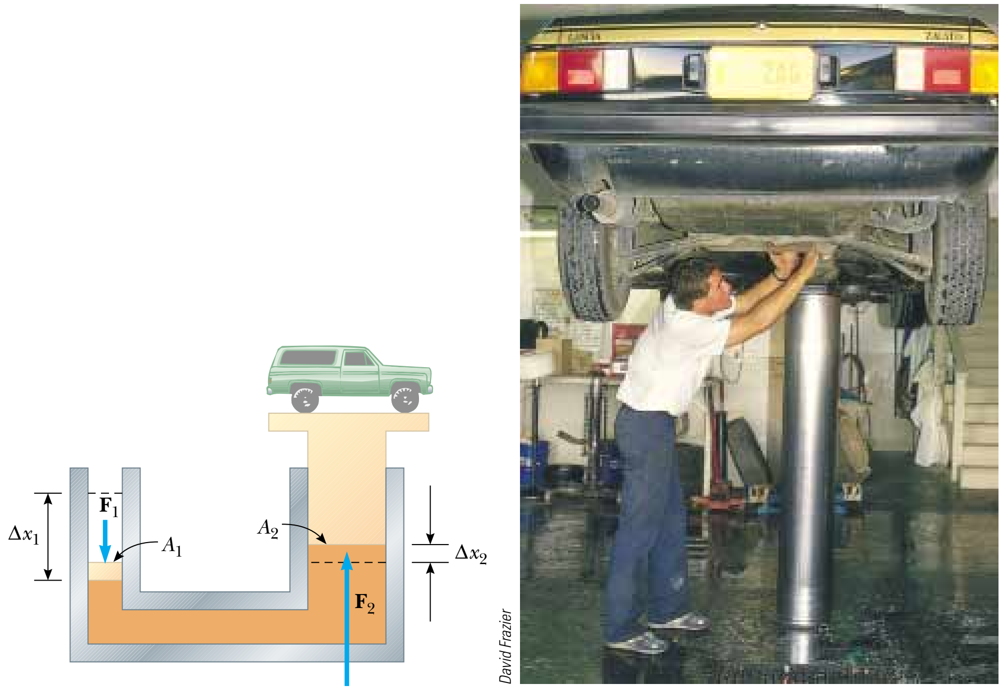
or

$$PA - P_0A - \rho Ahg = 0$$
$$PA - P_0A = \rho Ahg$$

Variation of pressure with depth

$$\boxed{P = P_0 + \rho gh} \qquad (14.4)$$

That is, **the pressure P at a depth h below a point in the liquid at which the pressure is P_0 is greater by an amount ρgh.** If the liquid is open to the atmosphere and P_0



(a) (b)

Figure 14.4 (a) Diagram of a hydraulic press. Because the increase in pressure is the same on the two sides, a small force $\mathbf{F}_1$ at the left produces a much greater force $\mathbf{F}_2$ at the right. (b) A vehicle undergoing repair is supported by a hydraulic lift in a garage.

is the pressure at the surface of the liquid, then P_0 is atmospheric pressure. In our calculations and working of end-of-chapter problems, we usually take atmospheric pressure to be

$$P_0 = 1.00 \text{ atm} = 1.013 \times 10^5 \text{ Pa}$$

Equation 14.4 implies that the pressure is the same at all points having the same depth, independent of the shape of the container.

In view of the fact that the pressure in a fluid depends on depth and on the value of P_0, any increase in pressure at the surface must be transmitted to every other point in the fluid. This concept was first recognized by the French scientist Blaise Pascal (1623–1662) and is called **Pascal's law: a change in the pressure applied to a fluid is transmitted undiminished to every point of the fluid and to the walls of the container.**

Pascal's law

An important application of Pascal's law is the hydraulic press illustrated in Figure 14.4a. A force of magnitude F_1 is applied to a small piston of surface area A_1. The pressure is transmitted through an incompressible liquid to a larger piston of surface area A_2. Because the pressure must be the same on both sides, $P = F_1/A_1 = F_2/A_2$. Therefore, the force F_2 is greater than the force F_1 by a factor A_2/A_1. By designing a hydraulic press with appropriate areas A_1 and A_2, a large output force can be applied by means of a small input force. Hydraulic brakes, car lifts, hydraulic jacks, and forklifts all make use of this principle (Fig. 14.4b).

Because liquid is neither added nor removed from the system, the volume of liquid pushed down on the left in Figure 14.4a as the piston moves downward through a displacement Δx_1 equals the volume of liquid pushed up on the right as the right piston moves upward through a displacement Δx_2. That is, $A_1 \, \Delta x_1 = A_2 \, \Delta x_2$; thus, $A_2/A_1 = \Delta x_1/\Delta x_2$. We have already shown that $A_2/A_1 = F_2/F_1$. Thus, $F_2/F_1 = \Delta x_1/\Delta x_2$, so $F_1 \, \Delta x_1 = F_2 \, \Delta x_2$. Each side of this equation is the work done by the force. Thus, the work done by $\mathbf{F}_1$ on the input piston equals the work done by $\mathbf{F}_2$ on the output piston, as it must in order to conserve energy.

Quick Quiz 14.2 The pressure at the bottom of a filled glass of water (ρ = 1 000 kg/m^3) is P. The water is poured out and the glass is filled with ethyl alcohol (ρ = 806 kg/m^3). The pressure at the bottom of the glass is (a) smaller than P (b) equal to P (c) larger than P (d) indeterminate.

Example 14.2 The Car Lift

Interactive

In a car lift used in a service station, compressed air exerts a force on a small piston that has a circular cross section and a radius of 5.00 cm. This pressure is transmitted by a liquid to a piston that has a radius of 15.0 cm. What force must the compressed air exert to lift a car weighing 13 300 N? What air pressure produces this force?

Solution Because the pressure exerted by the compressed air is transmitted undiminished throughout the liquid, we have

$$F_1 = \left(\frac{A_1}{A_2}\right) F_2 = \frac{\pi(5.00 \times 10^{-2}\text{ m})^2}{\pi(15.0 \times 10^{-2}\text{ m})^2}(1.33 \times 10^4\text{ N})$$

$$= \boxed{1.48 \times 10^3\text{ N}}$$

The air pressure that produces this force is

$$P = \frac{F_1}{A_1} = \frac{1.48 \times 10^3\text{ N}}{\pi(5.00 \times 10^{-2}\text{ m})^2}$$

$$= \boxed{1.88 \times 10^5\text{ Pa}}$$

This pressure is approximately twice atmospheric pressure.

You can adjust the weight of the truck in Figure 14.4a at the Interactive Worked Example link at **http://www.pse6.com.**

Example 14.3 A Pain in Your Ear

Estimate the force exerted on your eardrum due to the water above when you are swimming at the bottom of a pool that is 5.0 m deep.

Solution First, we must find the unbalanced pressure on the eardrum; then, after estimating the eardrum's surface area, we can determine the force that the water exerts on it.

The air inside the middle ear is normally at atmospheric pressure P_0. Therefore, to find the net force on the eardrum, we must consider the difference between the total pressure at the bottom of the pool and atmospheric pressure:

$$\begin{aligned} P_{\text{bot}} - P_0 &= \rho g h \\ &= (1.00 \times 10^3\text{ kg/m}^3)(9.80\text{ m/s}^2)(5.0\text{ m}) \\ &= 4.9 \times 10^4\text{ Pa} \end{aligned}$$

We estimate the surface area of the eardrum to be approximately 1 cm^2 = 1×10^{-4} m^2. This means that the force on it is $F = (P_{\text{bot}} - P_0)A \approx 5$ N. Because a force on the eardrum of this magnitude is extremely uncomfortable, swimmers often "pop their ears" while under water, an action that pushes air from the lungs into the middle ear. Using this technique equalizes the pressure on the two sides of the eardrum and relieves the discomfort.

Example 14.4 The Force on a Dam

Water is filled to a height H behind a dam of width w (Fig. 14.5). Determine the resultant force exerted by the water on the dam.

Solution Because pressure varies with depth, we cannot calculate the force simply by multiplying the area by the pressure. We can solve the problem by using Equation 14.2 to find the force dF exerted on a narrow horizontal strip at depth h and then integrating the expression to find the total force. Let us imagine a vertical y axis, with $y = 0$ at the bottom of the dam and our strip a distance y above the bottom.

We can use Equation 14.4 to calculate the pressure at the depth h; we omit atmospheric pressure because it acts on both sides of the dam:

$$P = \rho g h = \rho g(H - y)$$

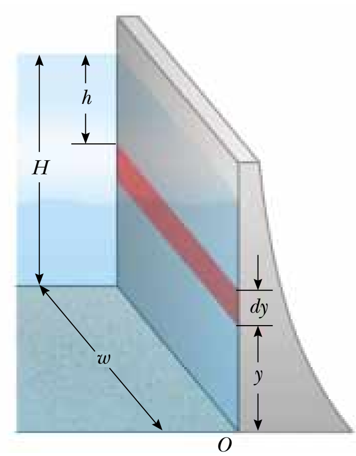


Figure 14.5 (Example 14.4)

Using Equation 14.2, we find that the force exerted on the shaded strip of area $dA = w\,dy$ is

$$dF = P\,dA = \rho g(H - y)w\,dy$$

Therefore, the total force on the dam is

$$F = \int P\,dA = \int_0^H \rho g(H - y)w\,dy = \boxed{\tfrac{1}{2}\rho g w H^2}$$

Note that the thickness of the dam shown in Figure 14.5 increases with depth. This design accounts for the greater and greater pressure that the water exerts on the dam at greater depths.

What If? **What if you were asked to find this force without using calculus? How could you determine its value?**

Answer We know from Equation 14.4 that the pressure varies linearly with depth. Thus, the average pressure due to the water over the face of the dam is the average of the pressure at the top and the pressure at the bottom:

$$P_{av} = \frac{P_{top} + P_{bottom}}{2} = \frac{0 + \rho g H}{2} = \tfrac{1}{2}\rho g H$$

Now, the total force is equal to the average pressure times the area of the face of the dam:

$$F = P_{av}A = (\tfrac{1}{2}\rho g H)(Hw) = \tfrac{1}{2}\rho g w H^2$$

which is the same result we obtained using calculus.

14.3 Pressure Measurements

During the weather report on a television news program, the *barometric pressure* is often provided. This is the current pressure of the atmosphere, which varies over a small range from the standard value provided earlier. How is this pressure measured?

One instrument used to measure atmospheric pressure is the common barometer, invented by Evangelista Torricelli (1608–1647). A long tube closed at one end is filled with mercury and then inverted into a dish of mercury (Fig. 14.6a). The closed end of the tube is nearly a vacuum, so the pressure at the top of the mercury column can be taken as zero. In Figure 14.6a, the pressure at point A, due to the column of mercury, must equal the pressure at point B, due to the atmosphere. If this were not the case, there would be a net force that would move mercury from one point to the other until equilibrium is established. Therefore, it follows that $P_0 = \rho_{Hg}gh$, where ρ_{Hg} is the density of the mercury and h is the height of the mercury column. As atmospheric pressure varies, the height of the mercury column varies, so the height can be calibrated to measure atmospheric pressure. Let us determine the height of a mercury column for one atmosphere of pressure, $P_0 = 1\text{ atm} = 1.013 \times 10^5$ Pa:

$$P_0 = \rho_{Hg}gh \longrightarrow h = \frac{P_0}{\rho_{Hg}g} = \frac{1.013 \times 10^5 \text{ Pa}}{(13.6 \times 10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)} = 0.760 \text{ m}$$

Based on a calculation such as this, one atmosphere of pressure is defined to be the pressure equivalent of a column of mercury that is exactly 0.760 0 m in height at 0°C.

A device for measuring the pressure of a gas contained in a vessel is the open-tube manometer illustrated in Figure 14.6b. One end of a U-shaped tube containing a liquid is open to the atmosphere, and the other end is connected to a system of unknown pressure P. The pressures at points A and B must be the same (otherwise, the curved portion of the liquid would experience a net force and would accelerate), and the pressure at A is the unknown pressure of the gas. Therefore, equating the unknown pressure P to the pressure at point B, we see that $P = P_0 + \rho gh$. The difference in pressure $P - P_0$ is equal to ρgh. The pressure P is called the **absolute pressure,** while the difference $P - P_0$ is called the **gauge pressure.** For example, the pressure you measure in your bicycle tire is gauge pressure.

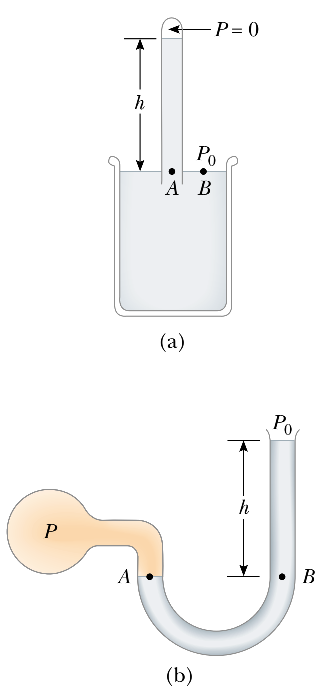


Figure 14.6 Two devices for measuring pressure: (a) a mercury barometer and (b) an open-tube manometer.

Quick Quiz 14.3 Several common barometers are built, with a variety of fluids. For which of the following fluids will the column of fluid in the barometer be the highest? (a) mercury (b) water (c) ethyl alcohol (d) benzene

Quick Quiz 14.4 You have invented a spacesuit with a straw passing through the faceplate so that you can drink from a glass while on the surface of a planet. Out on the surface of the Moon, you attempt to drink through the straw from an open glass of water. The value of g on the Moon is about one sixth of that on Earth. Compared to the difficulty in drinking through a straw on Earth, you find drinking through a straw on the Moon to be (a) easier (b) equally difficult (c) harder (d) impossible.

14.4 Buoyant Forces and Archimedes's Principle

Have you ever tried to push a beach ball under water (Fig. 14.7a)? This is extremely difficult to do because of the large upward force exerted by the water on the ball. The upward force exerted by a fluid on any immersed object is called a **buoyant force.** We can determine the magnitude of a buoyant force by applying some logic. Imagine a beach ball–sized parcel of water beneath the water surface, as in Figure 14.7b. Because this parcel is in equilibrium, there must be an upward force that balances the downward gravitational force on the parcel. This upward force is the buoyant force, and its magnitude is equal to the weight of the water in the parcel. The buoyant force is the resultant force due to all forces applied by the fluid surrounding the parcel.

Now imagine replacing the beach ball–sized parcel of water with a beach ball of the same size. The resultant force applied by the fluid surrounding the beach ball is the same, regardless of whether it is applied to a beach ball or to a parcel of water. Consequently, we can claim that **the magnitude of the buoyant force always equals the weight of the fluid displaced by the object.** This statement is known as **Archimedes's principle.**

With the beach ball under water, the buoyant force, equal to the weight of a beach ball-sized parcel of water, is much larger than the weight of the beach ball. Thus, there is a net upward force of large magnitude—this is why it is so hard to hold the beach ball under the water. Note that Archimedes's principle does not refer to the makeup of the object experiencing the buoyant force. The object's composition is not a factor in the buoyant force because the buoyant force is exerted by the fluid.

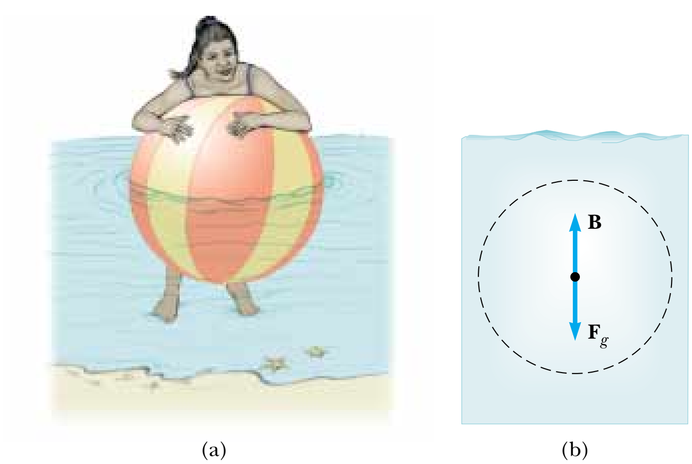


Figure 14.7. (a) A swimmer attempts to push a beach ball underwater. (b) The forces on a beach ball–sized parcel of water. The buoyant force **B** on a beach ball that replaces this parcel is exactly the same as the buoyant force on the parcel.

Archimedes

Greek Mathematician, Physicist, and Engineer (287–212 B.C.)

Archimedes was perhaps the greatest scientist of antiquity. He was the first to compute accurately the ratio of a circle's circumference to its diameter, and he also showed how to calculate the volume and surface area of spheres, cylinders, and other geometric shapes. He is well known for discovering the nature of the buoyant force and was also a gifted inventor. One of his practical inventions, still in use today, is Archimedes's screw, an inclined, rotating, coiled tube used originally to lift water from the holds of ships. He also invented the catapult and devised systems of levers, pulleys, and weights for raising heavy loads. Such inventions were successfully used to defend his native city, Syracuse, during a two-year siege by Romans.

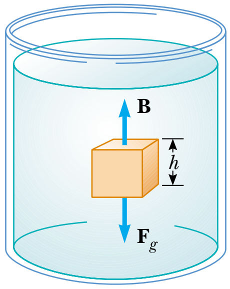


Figure 14.8 The external forces acting on the cube of liquid are the gravitational force $\mathbf{F}_g$ and the buoyant force $\mathbf{B}$. Under equilibrium conditions, $B = F_g$.

To understand the origin of the buoyant force, consider a cube immersed in a liquid as in Figure 14.8. The pressure P_b at the bottom of the cube is greater than the pressure P_t at the top by an amount $\rho_{\text{fluid}}gh$, where h is the height of the cube and ρ_{fluid} is the density of the fluid. The pressure at the bottom of the cube causes an *upward* force equal to P_bA, where A is the area of the bottom face. The pressure at the top of the cube causes a *downward* force equal to P_tA. The resultant of these two forces is the buoyant force $\mathbf{B}$:

$$B = (P_b - P_t)A = (\rho_{\text{fluid}}gh)A = \rho_{\text{fluid}}gV \tag{14.5}$$

where V is the volume of the fluid displaced by the cube. Because the product $\rho_{\text{fluid}}V$ is equal to the mass of fluid displaced by the object, we see that

$$B = Mg$$

where Mg is the weight of the fluid displaced by the cube. This is consistent with our initial statement about Archimedes's principle above, based on the discussion of the beach ball.

Before we proceed with a few examples, it is instructive for us to discuss two common situations—a totally submerged object and a floating (partly submerged) object.

PITFALL PREVENTION

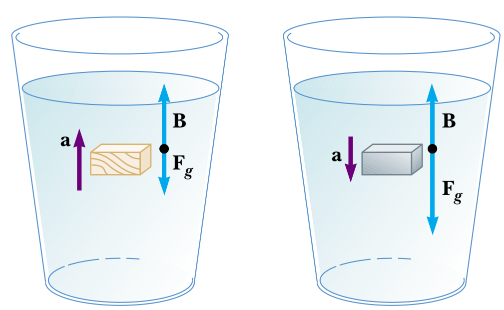
14.2 Buoyant Force is Exerted by the Fluid

Remember that **the buoyant force is exerted by the fluid.** It is not determined by properties of the object, except for the amount of fluid displaced by the object. Thus, if several objects of different densities but the same volume are immersed in a fluid, they will all experience the same buoyant force. Whether they sink or float will be determined by the relationship between the buoyant force and the weight.

Case 1: Totally Submerged Object When an object is totally submerged in a fluid of density ρ_{fluid}, the magnitude of the upward buoyant force is $B = \rho_{\text{fluid}}gV = \rho_{\text{fluid}}gV_{\text{obj}}$, where V_{obj} is the volume of the object. If the object has a mass M and density ρ_{obj}, its weight is equal to $F_g = Mg = \rho_{\text{obj}}gV_{\text{obj}}$, and the net force on it is $B - F_g = (\rho_{\text{fluid}} - \rho_{\text{obj}})gV_{\text{obj}}$. Hence, if the density of the object is less than the density of the fluid, then the downward gravitational force is less than the buoyant force, and the unsupported object accelerates upward (Fig. 14.9a). If the density of the object is greater than the density of the fluid, then the upward buoyant force is less than the downward gravitational force, and the unsupported object sinks (Fig. 14.9b). If the density of the submerged object equals the density of the fluid, the net force on the object is zero and it remains in equilibrium. Thus, **the direction of motion of an object submerged in a fluid is determined only by the densities of the object and the fluid.**

Case 2: Floating Object Now consider an object of volume V_{obj} and density $\rho_{\text{obj}} < \rho_{\text{fluid}}$ in static equilibrium floating on the surface of a fluid—that is, an object that is only *partially* submerged (Fig. 14.10). In this case, the upward buoyant force is balanced by the downward gravitational force acting on the object. If V_{fluid} is the volume of the fluid displaced by the object (this volume is the same as the volume of that part of the object that is beneath the surface of the fluid), the buoyant force has a magnitude $B = \rho_{\text{fluid}}gV_{\text{fluid}}$. Because the weight of the object is $F_g = Mg = \rho_{\text{obj}}gV_{\text{obj}}$, and because $F_g = B$, we see that $\rho_{\text{fluid}}gV_{\text{fluid}} = \rho_{\text{obj}}gV_{\text{obj}}$, or

$$\frac{V_{\text{fluid}}}{V_{\text{obj}}} = \frac{\rho_{\text{obj}}}{\rho_{\text{fluid}}} \tag{14.6}$$

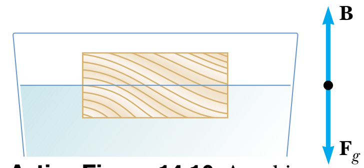


Active Figure 14.9 (a) A totally submerged object that is less dense than the fluid in which it is submerged experiences a net upward force. (b) A totally submerged object that is denser than the fluid experiences a net downward force and sinks.

At the Active Figures link at http://www.pse6.com, you can move the object to new positions as well as change the density of the object to see the results.

This equation tells us that **the fraction of the volume of a floating object that is below the fluid surface is equal to the ratio of the density of the object to that of the fluid.**

Under normal conditions, the weight of a fish is slightly greater than the buoyant force due to water. It follows that the fish would sink if it did not have some mechanism for adjusting the buoyant force. The fish accomplishes this by internally regulating the size of its air-filled swim bladder to increase its volume and the magnitude of the buoyant force acting on it. In this manner, fish are able to swim to various depths.



Active Figure 14.10 An object floating on the surface of a fluid experiences two forces, the gravitational force $\mathbf{F}_g$ and the buoyant force $\mathbf{B}$. Because the object floats in equilibrium, $B = F_g$.

At the Active Figures link at http://www.pse6.com, you can change the densities of the object and the fluid.

Quick Quiz 14.5 An apple is held completely submerged just below the surface of a container of water. The apple is then moved to a deeper point in the water. Compared to the force needed to hold the apple just below the surface, the force needed to hold it at a deeper point is (a) larger (b) the same (c) smaller (d) impossible to determine.

Quick Quiz 14.6 A glass of water contains a single floating ice cube (Fig. 14.11). When the ice melts, does the water level (a) go up (b) go down (c) remain the same?

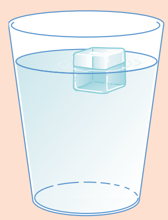


Figure 14.11 (Quick Quiz 14.6) An ice cube floats on the surface of water. What happens to the water level as the ice cube melts?

Quick Quiz 14.7 You are shipwrecked and floating in the middle of the ocean on a raft. Your cargo on the raft includes a treasure chest full of gold that you found before your ship sank, and the raft is just barely afloat. To keep you floating as high as possible in the water, should you (a) leave the treasure chest on top of the raft (b) secure the treasure chest to the underside of the raft (c) hang the treasure chest in the water with a rope attached to the raft? (Assume that throwing the treasure chest overboard is not an option you wish to consider!)

Example 14.5 Eureka!

Archimedes supposedly was asked to determine whether a crown made for the king consisted of pure gold. Legend has it that he solved this problem by weighing the crown first in air and then in water, as shown in Figure 14.12. Suppose the scale read 7.84 N in air and 6.84 N in water. What should Archimedes have told the king?

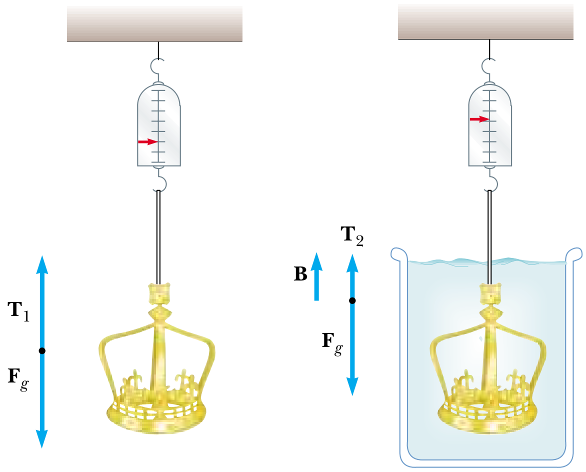
Solution Figure 14.12 helps us to conceptualize the problem. Because of our understanding of the buoyant force, we realize that the scale reading will be smaller in Figure 14.12b than in Figure 14.12a. The scale reading is a measure of one of the forces on the crown, and we recognize that the crown is stationary. Thus, we can categorize this as a force equilibrium problem. To analyze the problem, note that when the crown is suspended in air, the scale reads the true weight $T_1 = F_g$ (neglecting the buoyancy of air). When it is immersed in water, the buoyant force $\mathbf{B}$ reduces the scale reading to an *apparent* weight of $T_2 = F_g - B$. Because the crown is in equilibrium, the net force on it is zero. When the crown is in water,

$$\sum F = B + T_2 - F_g = 0$$

so that

$$B = F_g - T_2 = 7.84 \text{ N} - 6.84 \text{ N} = 1.00 \text{ N}$$

Because this buoyant force is equal in magnitude to the weight of the displaced water, we have $\rho_w g V_w = 1.00$ N, where V_w is the volume of the displaced water and ρ_w is its



(a) (b)

Figure 14.12 (Example 14.5) (a) When the crown is suspended in air, the scale reads its true weight because $T_1 = F_g$ (the buoyancy of air is negligible). (b) When the crown is immersed in water, the buoyant force $\mathbf{B}$ changes the scale reading to a lower value $T_2 = F_g - B$.

density. Also, the volume of the crown V_c is equal to the volume of the displaced water because the crown is completely submerged. Therefore,

$$V_c = V_w = \frac{1.00\ \text{N}}{\rho_w g} = \frac{1.00\ \text{N}}{(1\ 000\ \text{kg/m}^3)(9.80\ \text{m/s}^2)}$$
$$= 1.02 \times 10^{-4}\ \text{m}^3$$

Finally, the density of the crown is

$$\rho_c = \frac{m_c}{V_c} = \frac{m_c g}{V_c g} = \frac{7.84\ \text{N}}{(1.02 \times 10^{-4}\ \text{m}^3)(9.80\ \text{m/s}^2)}$$
$$= 7.84 \times 10^3\ \text{kg/m}^3$$

To finalize the problem, from Table 14.1 we see that the density of gold is $19.3 \times 10^3\ \text{kg/m}^3$. Thus, Archimedes should have told the king that he had been cheated. Either the crown was hollow, or it was not made of pure gold.

What If? **Suppose the crown has the same weight but were indeed pure gold and not hollow. What would the scale reading be when the crown is immersed in water?**

Answer We first find the volume of the solid gold crown:

$$V_c = \frac{m_c}{\rho_c} = \frac{m_c g}{\rho_c g} = \frac{7.84\ \text{N}}{(19.3 \times 10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)}$$
$$= 4.15 \times 10^{-5}\ \text{m}^3$$

Now, the buoyant force on the crown will be

$$B = \rho_w g V_w = \rho_w g V_c$$
$$= (1.00 \times 10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(4.15 \times 10^{-5}\ \text{m}^3)$$
$$= 0.406\ \text{N}$$

and the tension in the string hanging from the scale is

$$T_2 = F_g - B = 7.84\ \text{N} - 0.406\ \text{N} = 7.43\ \text{N}$$

Example 14.6 A Titanic Surprise

An iceberg floating in seawater, as shown in Figure 14.13a, is extremely dangerous because most of the ice is below the surface. This hidden ice can damage a ship that is still a considerable distance from the visible ice. What fraction of the iceberg lies below the water level?

Solution This problem corresponds to Case 2. The weight of the iceberg is $F_g = \rho_i V_i g$, where $\rho_i = 917\ \text{kg/m}^3$ and V_i is the volume of the whole iceberg. The magnitude of the upward buoyant force equals the weight of the displaced water: $B = \rho_w V_w g$, where V_w, the volume of the displaced water, is equal to the volume of the ice beneath the water (the shaded region in Fig. 14.13b) and ρ_w is the density of seawater, $\rho_w = 1\ 030\ \text{kg/m}^3$. Because $\rho_i V_i g = \rho_w V_w g$, the fraction of ice beneath the water's surface is

$$f = \frac{V_w}{V_i} = \frac{\rho_i}{\rho_w} = \frac{917\ \text{kg/m}^3}{1\ 030\ \text{kg/m}^3} = \boxed{0.890} \text{ or } \boxed{89.0\%}$$

(a) (b)

Figure 14.13 (Example 14.6) (a) Much of the volume of this iceberg is beneath the water. (b) A ship can be damaged even when it is not near the visible ice.


Andy Sacks/Getty

Figure 14.14 Laminar flow around an automobile in a test wind tunnel.

14.5 Fluid Dynamics

Thus far, our study of fluids has been restricted to fluids at rest. We now turn our attention to fluids in motion. When fluid is in motion, its flow can be characterized as being one of two main types. The flow is said to be **steady,** or **laminar,** if each particle of the fluid follows a smooth path, such that the paths of different particles never cross each other, as shown in Figure 14.14. In steady flow, the velocity of fluid particles passing any point remains constant in time.

Above a certain critical speed, fluid flow becomes **turbulent;** turbulent flow is irregular flow characterized by small whirlpool-like regions, as shown in Figure 14.15.

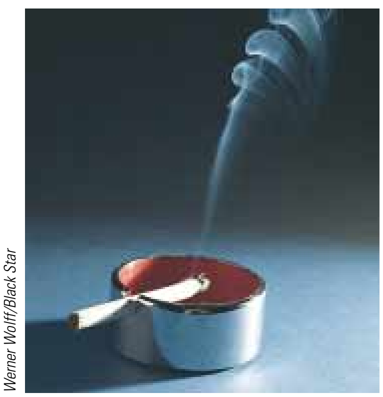

Werner Wolff/Black Star

Figure 14.15 Hot gases from a cigarette made visible by smoke particles. The smoke first moves in laminar flow at the bottom and then in turbulent flow above.

The term **viscosity** is commonly used in the description of fluid flow to characterize the degree of internal friction in the fluid. This internal friction, or *viscous force,* is associated with the resistance that two adjacent layers of fluid have to moving relative to each other. Viscosity causes part of the kinetic energy of a fluid to be converted to internal energy. This mechanism is similar to the one by which an object sliding on a rough horizontal surface loses kinetic energy.

Because the motion of real fluids is very complex and not fully understood, we make some simplifying assumptions in our approach. In our model of **ideal fluid flow,** we make the following four assumptions:

1. **The fluid is nonviscous.** In a nonviscous fluid, internal friction is neglected. An object moving through the fluid experiences no viscous force.
2. **The flow is steady.** In steady (laminar) flow, the velocity of the fluid at each point remains constant.
3. **The fluid is incompressible.** The density of an incompressible fluid is constant.
4. **The flow is irrotational.** In irrotational flow, the fluid has no angular momentum about any point. If a small paddle wheel placed anywhere in the fluid does not rotate about the wheel's center of mass, then the flow is irrotational.

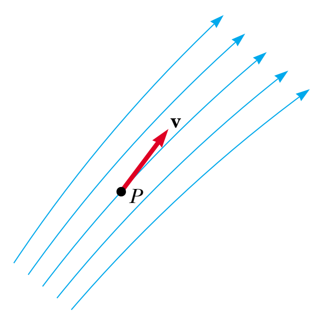


Figure 14.16 A particle in laminar flow follows a streamline, and at each point along its path the particle's velocity is tangent to the streamline.

The path taken by a fluid particle under steady flow is called a **streamline.** The velocity of the particle is always tangent to the streamline, as shown in Figure 14.16. A set of streamlines like the ones shown in Figure 14.16 form a *tube of flow.* Note that fluid

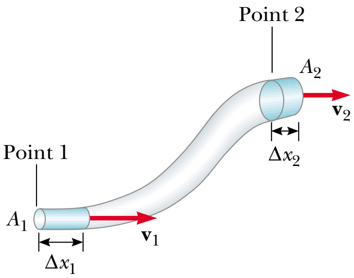


Figure 14.17 A fluid moving with steady flow through a pipe of varying cross-sectional area. The volume of fluid flowing through area A_1 in a time interval Δt must equal the volume flowing through area A_2 in the same time interval. Therefore, $A_1v_1 = A_2v_2$.

particles cannot flow into or out of the sides of this tube; if they could, then the streamlines would cross each other.

Consider an ideal fluid flowing through a pipe of nonuniform size, as illustrated in Figure 14.17. The particles in the fluid move along streamlines in steady flow. In a time interval Δt, the fluid at the bottom end of the pipe moves a distance $\Delta x_1 = v_1 \Delta t$. If A_1 is the cross-sectional area in this region, then the mass of fluid contained in the left shaded region in Figure 14.17 is $m_1 = \rho A_1 \Delta x_1 = \rho A_1 v_1 \Delta t$, where ρ is the (unchanging) density of the ideal fluid. Similarly, the fluid that moves through the upper end of the pipe in the time interval Δt has a mass $m_2 = \rho A_2 v_2 \Delta t$. However, because the fluid is incompressible and because the flow is steady, the mass that crosses A_1 in a time interval Δt must equal the mass that crosses A_2 in the same time interval. That is, $m_1 = m_2$, or $\rho A_1 v_1 = \rho A_2 v_2$; this means that

$$A_1 v_1 = A_2 v_2 = \text{constant} \tag{14.7}$$

This expression is called the **equation of continuity for fluids.** It states that

the product of the area and the fluid speed at all points along a pipe is constant for an incompressible fluid.

Equation 14.7 tells us that the speed is high where the tube is constricted (small A) and low where the tube is wide (large A). The product Av, which has the dimensions of volume per unit time, is called either the *volume flux* or the *flow rate*. The condition $Av =$ constant is equivalent to the statement that the volume of fluid that enters one end of a tube in a given time interval equals the volume leaving the other end of the tube in the same time interval if no leaks are present.

You demonstrate the equation of continuity each time you water your garden with your thumb over the end of a garden hose as in Figure 14.18. By partially blocking the opening with your thumb, you reduce the cross-sectional area through which the water passes. As a result, the speed of the water increases as it exits the hose, and it can be sprayed over a long distance.

Quick Quiz 14.8 You tape two different soda straws together end-to-end to make a longer straw with no leaks. The two straws have radii of 3 mm and 5 mm. You drink a soda through your combination straw. In which straw is the speed of the liquid the highest? (a) whichever one is nearest your mouth (b) the one of radius 3 mm (c) the one of radius 5 mm (d) Neither—the speed is the same in both straws.

George Semple

Figure 14.18 The speed of water spraying from the end of a garden hose increases as the size of the opening is decreased with the thumb.

Example 14.7 Niagara Falls

Each second, 5 525 m^3 of water flows over the 670-m-wide cliff of the Horseshoe Falls portion of Niagara Falls. The water is approximately 2 m deep as it reaches the cliff. What is its speed at that instant?

Solution The cross-sectional area of the water as it reaches the edge of the cliff is $A = (670\text{ m})(2\text{ m}) = 1\,340\text{ m}^2$. The flow rate of 5 525 m^3/s is equal to Av. This gives

$$v = \frac{5\,525\text{ m}^3/\text{s}}{A} = \frac{5\,525\text{ m}^3/\text{s}}{1\,340\text{ m}^2} = \boxed{4\text{ m/s}}$$

Note that we have kept only one significant figure because our value for the depth has only one significant figure.

Example 14.8 Watering a Garden

A water hose 2.50 cm in diameter is used by a gardener to fill a 30.0-L bucket. The gardener notes that it takes 1.00 min to fill the bucket. A nozzle with an opening of cross-sectional area 0.500 cm^2 is then attached to the hose. The nozzle is held so that water is projected horizontally from a point 1.00 m above the ground. Over what horizontal distance can the water be projected?

Solution We identify point 1 within the hose and point 2 at the exit of the nozzle. We first find the speed of the water in the hose from the bucket-filling information. The cross-sectional area of the hose is

$$A_1 = \pi r^2 = \pi\frac{d^2}{4} = \pi\left(\frac{(2.50\text{ cm})^2}{4}\right) = 4.91\text{ cm}^2$$

According to the data given, the volume flow rate is equal to 30.0 L/min:

$$A_1 v_1 = 30.0\text{ L/min} = \frac{30.0 \times 10^3\text{ cm}^3}{60.0\text{ s}} = 500\text{ cm}^3/\text{s}$$

$$v_1 = \frac{500\text{ cm}^3/\text{s}}{A_1} = \frac{500\text{ cm}^3/\text{s}}{4.91\text{ cm}^2} = 102\text{ cm/s} = 1.02\text{ m/s}$$

Now we use the continuity equation for fluids to find the speed $v_2 = v_{xi}$ with which the water exits the nozzle. The subscript i anticipates that this will be the *initial* velocity component of the water projected from the hose, and the subscript x recognizes that the initial velocity vector of the projected water is in the horizontal direction.

$$A_1 v_1 = A_2 v_2 = A_2 v_{xi} \longrightarrow v_{xi} = \frac{A_1}{A_2}v_1$$

$$v_{xi} = \frac{4.91\text{ cm}^2}{0.500\text{ cm}^2}(1.02\text{ m/s})$$
$$= 10.0\text{ m/s}$$

We now shift our thinking away from fluids and to projectile motion because the water is in free fall once it exits the nozzle. A particle of the water falls through a vertical distance of 1.00 m starting from rest, and lands on the ground at a time that we find from Equation 2.12:

$$y_f = y_i + v_{yi}t - \tfrac{1}{2}gt^2$$

$$-1.00\text{ m} = 0 + 0 - \tfrac{1}{2}(9.80\text{ m/s}^2)t^2$$

$$t = \sqrt{\frac{2(1.00\text{ m})}{9.80\text{ m/s}^2}} = 0.452\text{ s}$$

In the horizontal direction, we apply Equation 2.12 with $a_x = 0$ to a particle of water to find the horizontal distance:

$$x_f = x_i + v_{xi}t = 0 + (10.0\text{ m/s})(0.452\text{ s}) = \boxed{4.52\text{ m}}$$

14.6 Bernoulli's Equation

You have probably had the experience of driving on a highway and having a large truck pass you at high speed. In this situation, you may have had the frightening feeling that your car was being pulled in toward the truck as it passed. We will investigate the origin of this effect in this section.

As a fluid moves through a region where its speed and/or elevation above the Earth's surface changes, the pressure in the fluid varies with these changes. The relationship between fluid speed, pressure, and elevation was first derived in 1738 by the Swiss physicist Daniel Bernoulli. Consider the flow of a segment of an ideal fluid through a nonuniform pipe in a time interval Δt, as illustrated in Figure 14.19. At the beginning of the time interval, the segment of fluid consists of the blue shaded portion (portion 1) at the left and the unshaded portion. During the time interval, the left end of the segment moves to the right by a distance Δx_1, which is the length of the blue shaded portion at the left. Meanwhile, the right end of the segment moves to the right through a distance Δx_2, which is the length of the blue shaded portion (portion 2) at the upper right of Figure 14.19.

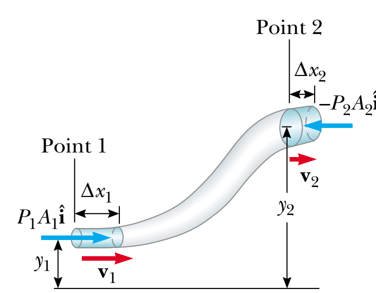


Figure 14.19 A fluid in laminar flow through a constricted pipe. The volume of the shaded portion on the left is equal to the volume of the shaded portion on the right.

Daniel Bernoulli

Swiss physicist
(1700–1782)

Daniel Bernoulli made important discoveries in fluid dynamics. Born into a family of mathematicians, he was the only member of the family to make a mark in physics.

Bernoulli's most famous work, *Hydrodynamica,* was published in 1738; it is both a theoretical and a practical study of equilibrium, pressure, and speed in fluids. He showed that as the speed of a fluid increases, its pressure decreases. Referred to as "Bernoulli's principle," his work is used to produce a partial vacuum in chemical laboratories by connecting a vessel to a tube through which water is running rapidly.

In *Hydrodynamica,* Bernoulli also attempted the first explanation of the behavior of gases with changing pressure and temperature; this was the beginning of the kinetic theory of gases, a topic we study in Chapter 21. *(Corbis–Bettmann)*

Thus, at the end of the time interval, the segment of fluid consists of the unshaded portion and the blue shaded portion at the upper right.

Now consider forces exerted on this segment by fluid to the left and the right of the segment. The force exerted by the fluid on the left end has a magnitude P_1A_1. The work done by this force on the segment in a time interval Δt is $W_1 = F_1\,\Delta x_1 = P_1A_1\,\Delta x_1 = P_1V$, where V is the volume of portion 1. In a similar manner, the work done by the fluid to the right of the segment in the same time interval Δt is $W_2 = -P_2A_2\,\Delta x_2 = -P_2V$. (The volume of portion 1 equals the volume of portion 2.) This work is negative because the force on the segment of fluid is to the left and the displacement is to the right. Thus, the net work done on the segment by these forces in the time interval Δt is

$$W = (P_1 - P_2)V$$

Part of this work goes into changing the kinetic energy of the segment of fluid, and part goes into changing the gravitational potential energy of the segment–Earth system. Because we are assuming streamline flow, the kinetic energy of the unshaded portion of the segment in Figure 14.19 is unchanged during the time interval. The only change is as follows: before the time interval we have portion 1 traveling at v_1, whereas after the time interval, we have portion 2 traveling at v_2. Thus, the change in the kinetic energy of the segment of fluid is

$$\Delta K = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2$$

where m is the mass of both portion 1 and portion 2. (Because the volumes of both portions are the same, they also have the same mass.)

Considering the gravitational potential energy of the segment–Earth system, once again there is no change during the time interval for the unshaded portion of the fluid. The net change is that the mass of the fluid in portion 1 has effectively been moved to the location of portion 2. Consequently, the change in gravitational potential energy is

$$\Delta U = mgy_2 - mgy_1$$

The total work done on the system by the fluid outside the segment is equal to the change in mechanical energy of the system: $W = \Delta K + \Delta U$. Substituting for each of these terms, we obtain

$$(P_1 - P_2)V = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2 + mgy_2 - mgy_1$$

If we divide each term by the portion volume V and recall that $\rho = m/V$, this expression reduces to

$$P_1 - P_2 = \tfrac{1}{2}\rho v_2^2 - \tfrac{1}{2}\rho v_1^2 + \rho gy_2 - \rho gy_1$$

Rearranging terms, we obtain

$$P_1 + \tfrac{1}{2}\rho v_1^2 + \rho gy_1 = P_2 + \tfrac{1}{2}\rho v_2^2 + \rho gy_2 \qquad (14.8)$$

This is **Bernoulli's equation** as applied to an ideal fluid. It is often expressed as

Bernoulli's equation

$$\boxed{P + \tfrac{1}{2}\rho v^2 + \rho gy = \text{constant}} \qquad (14.9)$$

This expression shows that the pressure of a fluid decreases as the speed of the fluid increases. In addition, the pressure decreases as the elevation increases. This explains why water pressure from faucets on the upper floors of a tall building is weak unless measures are taken to provide higher pressure for these upper floors.

When the fluid is at rest, $v_1 = v_2 = 0$ and Equation 14.8 becomes

$$P_1 - P_2 = \rho g(y_2 - y_1) = \rho gh$$

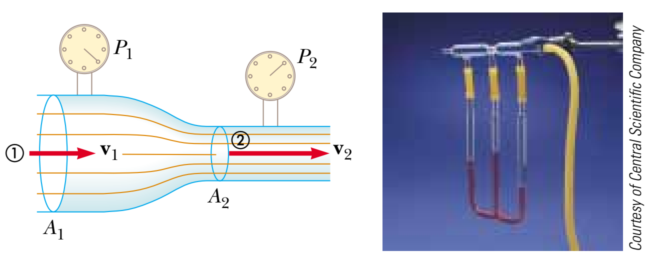
This is in agreement with Equation 14.4.

While Equation 14.9 was derived for an incompressible fluid, the general behavior of pressure with speed is true even for gases—as the speed increases, the pressure decreases. This *Bernoulli effect* explains the experience with the truck on the highway at the opening of this section. As air passes between you and the truck, it must pass through a relatively narrow channel. According to the continuity equation, the speed of the air is higher. According to the Bernoulli effect, this higher speed air exerts less pressure on your car than the slower moving air on the other side of your car. Thus, there is a net force pushing you toward the truck!

Quick Quiz 14.9 You observe two helium balloons floating next to each other at the ends of strings secured to a table. The facing surfaces of the balloons are separated by 1–2 cm. You blow through the small space between the balloons. What happens to the balloons? (a) They move toward each other. (b) They move away from each other. (c) They are unaffected.

Example 14.9 The Venturi Tube

The horizontal constricted pipe illustrated in Figure 14.20, known as a *Venturi tube*, can be used to measure the flow speed of an incompressible fluid. Determine the flow speed at point 2 if the pressure difference $P_1 - P_2$ is known.



(a) (b)

Figure 14.20 (Example 14.9) (a) Pressure P_1 is greater than pressure P_2 because $v_1 < v_2$. This device can be used to measure the speed of fluid flow. (b) A Venturi tube, located at the top of the photograph. The higher level of fluid in the middle column shows that the pressure at the top of the column, which is in the constricted region of the Venturi tube, is lower.

Solution Because the pipe is horizontal, $y_1 = y_2$, and applying Equation 14.8 to points 1 and 2 gives

$$(1) \qquad P_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \tfrac{1}{2}\rho v_2^2$$

From the equation of continuity, $A_1 v_1 = A_2 v_2$, we find that

$$(2) \qquad v_1 = \frac{A_2}{A_1} v_2$$

Substituting this expression into Equation (1) gives

$$P_1 + \tfrac{1}{2}\rho\left(\frac{A_2}{A_1}\right)^2 v_2^2 = P_2 + \tfrac{1}{2}\rho v_2^2$$

$$v_2 = \boxed{A_1\sqrt{\frac{2(P_1 - P_2)}{\rho(A_1^2 - A_2^2)}}}$$

We can use this result and the continuity equation to obtain an expression for v_1. Because $A_2 < A_1$, Equation (2) shows us that $v_2 > v_1$. This result, together with Equation (1), indicates that $P_1 > P_2$. In other words, the pressure is reduced in the constricted part of the pipe.

Example 14.10 Torricelli's Law

Interactive

An enclosed tank containing a liquid of density ρ has a hole in its side at a distance y_1 from the tank's bottom (Fig. 14.21). The hole is open to the atmosphere, and its diameter is much smaller than the diameter of the tank. The air above the liquid is maintained at a pressure P. Determine the speed of the liquid as it leaves the hole when the liquid's level is a distance h above the hole.

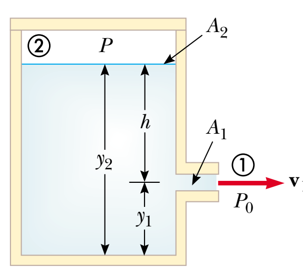


Figure 14.21 (Example 14.10) A liquid leaves a hole in a tank at speed v_1.

Solution Because $A_2 \gg A_1$, the liquid is approximately at rest at the top of the tank, where the pressure is P. Applying Bernoulli's equation to points 1 and 2 and noting that at the hole P_1 is equal to atmospheric pressure P_0, we find that

$$P_0 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 = P + \rho g y_2$$

But $y_2 - y_1 = h$; thus, this expression reduces to

$$v_1 = \sqrt{\frac{2(P - P_0)}{\rho} + 2gh}$$

When P is much greater than P_0 (so that the term $2gh$ can be neglected), the exit speed of the water is mainly a function of P. If the tank is open to the atmosphere, then $P = P_0$ and $v_1 = \sqrt{2gh}$. In other words, for an open tank, the speed of liquid coming out through a hole a distance h below the surface is equal to that acquired by an object falling freely through a vertical distance h. This phenomenon is known as **Torricelli's law.**

What If? What if the position of the hole in Figure 14.21 could be adjusted vertically? If the tank is open to the atmosphere and sitting on a table, what position of the hole would cause the water to land on the table at the farthest distance from the tank?

Answer We model a parcel of water exiting the hole as a projectile. We find the time at which the parcel strikes the table from a hole at an arbitrary position:

$$y_f = y_i + v_{yi}t - \tfrac{1}{2}gt^2$$

$$0 = y_1 + 0 - \tfrac{1}{2}gt^2$$

$$t = \sqrt{\frac{2y_1}{g}}$$

Thus, the horizontal position of the parcel at the time it strikes the table is

$$x_f = x_i + v_{xi}t = 0 + \sqrt{2g(y_2 - y_1)}\sqrt{\frac{2y_1}{g}}$$

$$= 2\sqrt{(y_2y_1 - y_1^2)}$$

Now we maximize the horizontal position by taking the derivative of x_f with respect to y_1 (because y_1, the height of the hole, is the variable that can be adjusted) and setting it equal to zero:

$$\frac{dx_f}{dy_1} = \tfrac{1}{2}(2)(y_2y_1 - y_1^2)^{-1/2}(y_2 - 2y_1) = 0$$

This is satisfied if

$$y_1 = \tfrac{1}{2}y_2$$

Thus, the hole should be halfway between the bottom of the tank and the upper surface of the water to maximize the horizontal distance. Below this location, the water is projected at a higher speed, but falls for a short time interval, reducing the horizontal range. Above this point, the water is in the air for a longer time interval, but is projected with a smaller horizontal speed.

At the Interactive Worked Example link at http://www.pse6.com, *you can move the hole vertically to see where the water lands.*

14.7 Other Applications of Fluid Dynamics

Consider the streamlines that flow around an airplane wing as shown in Figure 14.22. Let us assume that the airstream approaches the wing horizontally from the right with a velocity $\mathbf{v}_1$. The tilt of the wing causes the airstream to be deflected downward with a velocity $\mathbf{v}_2$. Because the airstream is deflected by the wing, the wing must exert a force on the airstream. According to Newton's third law, the airstream exerts a force $\mathbf{F}$ on the wing that is equal in magnitude and opposite in direction. This force has a vertical component called the **lift** (or aerodynamic lift) and a horizontal component called **drag.** The lift depends on several factors, such as the speed of the airplane, the area of the wing, its curvature, and the angle between the wing and the horizontal. The curvature of the wing surfaces causes the pressure above the wing to be lower than that below the wing, due to the Bernoulli effect. This assists with the lift on the wing. As the angle between the wing and the horizontal increases, turbulent flow can set in above the wing to reduce the lift.

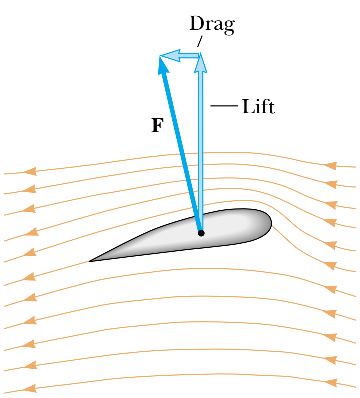


Figure 14.22 Streamline flow around a moving airplane wing. The air approaching from the right is deflected downward by the wing. By Newton's third law, this must coincide with an upward force on the wing from the air—*lift*. Because of air resistance, there is also a force opposite the velocity of the wing—*drag*.

In general, an object moving through a fluid experiences lift as the result of any effect that causes the fluid to change its direction as it flows past the object. Some factors that influence lift are the shape of the object, its orientation with respect to the fluid flow, any spinning motion it might have, and the texture of its surface. For example, a golf ball struck with a club is given a rapid backspin due to the slant of the club. The dimples on the ball increase the friction force between the ball and the air so that air adheres to the ball's surface. This effect is most pronounced on the top half of the ball, where the ball's surface is moving in the same direction as the air flow. Figure 14.23 shows air adhering to the ball and being deflected downward as a result. Because the ball pushes the air down, the air must push up on the ball. Without the dimples, the friction force is lower, and the golf ball does not travel as far. It may seem counterintuitive to increase the range by increasing the friction force, but the lift gained by spinning the ball more than compensates for the loss of range due to the effect of friction

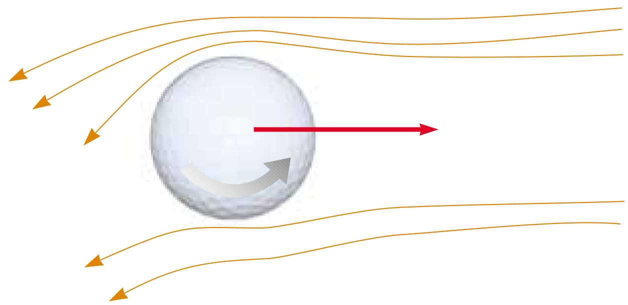


Figure 14.23 Because of the deflection of air, a spinning golf ball experiences a lifting force that allows it to travel much farther than it would if it were not spinning.

on the translational motion of the ball! For the same reason, a baseball's cover helps the spinning ball "grab" the air rushing by and helps to deflect it when a "curve ball" is thrown.

A number of devices operate by means of the pressure differentials that result from differences in a fluid's speed. For example, a stream of air passing over one end of an open tube, the other end of which is immersed in a liquid, reduces the pressure above the tube, as illustrated in Figure 14.24. This reduction in pressure causes the liquid to rise into the air stream. The liquid is then dispersed into a fine spray of droplets. You might recognize that this so-called atomizer is used in perfume bottles and paint sprayers.

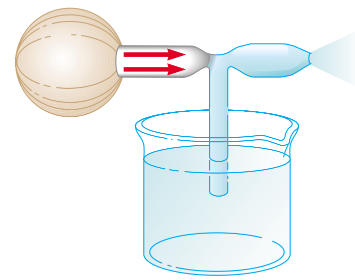


Figure 14.24 A stream of air passing over a tube dipped into a liquid causes the liquid to rise in the tube.

SUMMARY

Take a practice test for this chapter by clicking on the Practice Test link at http://www.pse6.com.

The **pressure** P in a fluid is the force per unit area exerted by the fluid on a surface:

$$P \equiv \frac{F}{A} \tag{14.1}$$

In the SI system, pressure has units of newtons per square meter (N/m^2), and 1 N/m^2 = 1 **pascal** (Pa).

The pressure in a fluid at rest varies with depth h in the fluid according to the expression

$$P = P_0 + \rho g h \tag{14.4}$$

where P_0 is the pressure at $h = 0$ and ρ is the density of the fluid, assumed uniform.

Pascal's law states that when pressure is applied to an enclosed fluid, the pressure is transmitted undiminished to every point in the fluid and to every point on the walls of the container.

When an object is partially or fully submerged in a fluid, the fluid exerts on the object an upward force called the **buoyant force.** According to **Archimedes's principle,** the magnitude of the buoyant force is equal to the weight of the fluid displaced by the object:

$$B = \rho_{\text{fluid}} g V \tag{14.5}$$

You can understand various aspects of a fluid's dynamics by assuming that the fluid is nonviscous and incompressible, and that the fluid's motion is a steady flow with no rotation.

Two important concepts regarding ideal fluid flow through a pipe of nonuniform size are as follows:

1. The flow rate (volume flux) through the pipe is constant; this is equivalent to stating that the product of the cross-sectional area A and the speed v at any point is a constant. This result is expressed in the **equation of continuity for fluids:**

$$A_1 v_1 = A_2 v_2 = \text{constant} \tag{14.7}$$

2. The sum of the pressure, kinetic energy per unit volume, and gravitational potential energy per unit volume has the same value at all points along a streamline. This result is summarized in **Bernoulli's equation:**

$$P + \tfrac{1}{2}\rho v^2 + \rho g y = \text{constant} \tag{14.9}$$

QUESTIONS

1. Two drinking glasses having equal weights but different shapes and different cross-sectional areas are filled to the same level with water. According to the expression $P = P_0 + \rho g h$, the pressure is the same at the bottom of both glasses. In view of this, why does one weigh more than the other?

2. Figure Q14.2 shows aerial views from directly above two dams. Both dams are equally wide (the vertical dimension in the diagram) and equally high (into the page in the diagram). The dam on the left holds back a very large lake, while the dam on the right holds back a narrow river. Which dam has to be built stronger?

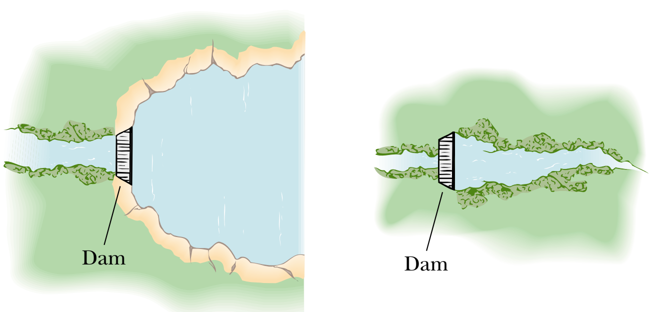


Figure Q14.2

3. Some physics students attach a long tube to the opening of a hot water bottle made of strong rubber. Leaving the hot water bottle on the ground, they hoist the other end of the tube to the roof of a multistory campus building. Students at the top of the building pour water into the tube. The students on the ground watch the bottle fill with water. On the roof, the students are surprised to see that the tube never seems to fill up—they can continue to pour more and more water down the tube. On the ground, the hot water bottle swells up like a balloon and bursts, drenching the students. Explain these observations.

4. If the top of your head has a surface area of 100 cm^2, what is the weight of the air above your head?

5. A helium-filled balloon rises until its density becomes the same as that of the surrounding air. If a sealed submarine begins to sink, will it go all the way to the bottom of the ocean or will it stop when its density becomes the same as that of the surrounding water?

6. A fish rests on the bottom of a bucket of water while the bucket is being weighed on a scale. When the fish begins to swim around, does the scale reading change?

7. Will a ship ride higher in the water of an inland lake or in the ocean? Why?

8. Suppose a damaged ship can just barely keep afloat in the ocean. It is towed toward shore and into a river, heading toward a dry dock for repair. As it is pulled up the river, it sinks. Why?

9. Lead has a greater density than iron, and both are denser than water. Is the buoyant force on a lead object greater than, less than, or equal to the buoyant force on an iron object of the same volume?

10. The water supply for a city is often provided from reservoirs built on high ground. Water flows from the reservoir, through pipes, and into your home when you turn the tap on your faucet. Why is the water flow more rapid out of a faucet on the first floor of a building than in an apartment on a higher floor?

11. Smoke rises in a chimney faster when a breeze is blowing. Use the Bernoulli effect to explain this phenomenon.

12. If the air stream from a hair dryer is directed over a Ping-Pong ball, the ball can be levitated. Explain.

13. When ski jumpers are airborne (Fig. Q14.13), why do they bend their bodies forward and keep their hands at their sides?

Figure Q14.13

14. When an object is immersed in a liquid at rest, why is the net force on the object in the horizontal direction equal to zero?

15. Explain why a sealed bottle partially filled with a liquid can float in a basin of the same liquid.

16. When is the buoyant force on a swimmer greater—after exhaling or after inhaling?

17. A barge is carrying a load of gravel along a river. It approaches a low bridge and the captain realizes that the top of the pile of gravel is not going to make it under the bridge. The captain orders the crew to quickly shovel gravel from the pile into the water. Is this a good decision?

18. A person in a boat floating in a small pond throws an anchor overboard. Does the level of the pond rise, fall, or remain the same?

19. An empty metal soap dish barely floats in water. A bar of Ivory soap floats in water. When the soap is stuck in the soap dish, the combination sinks. Explain why.

20. A piece of unpainted porous wood barely floats in a container partly filled with water. If the container is sealed and pressurized above atmospheric pressure, does the wood rise, fall, or remain at the same level?

21. A flat plate is immersed in a liquid at rest. For what orientation of the plate is the pressure on its flat surface uniform?

22. Because atmospheric pressure is about 10^5 N/m^2 and the area of a person's chest is about 0.13 m^2, the force of the atmosphere on one's chest is around 13 000 N. In view of this enormous force, why don't our bodies collapse?

23. How would you determine the density of an irregularly shaped rock?

24. Why do airplane pilots prefer to take off into the wind?

25. If you release a ball while inside a freely falling elevator, the ball remains in front of you rather than falling to the floor, because the ball, the elevator, and you all experience the same downward acceleration **g**. What happens if you repeat this experiment with a helium-filled balloon? (This one is tricky.)

26. Two identical ships set out to sea. One is loaded with a cargo of Styrofoam, and the other is empty. Which ship is more submerged?

27. A small piece of steel is tied to a block of wood. When the wood is placed in a tub of water with the steel on top, half of the block is submerged. If the block is inverted so that the steel is under water, does the amount of the block submerged increase, decrease, or remain the same? What happens to the water level in the tub when the block is inverted?

28. Prairie dogs (Fig. Q14.28) ventilate their burrows by building a mound around one entrance, which is open to a stream of air when wind blows from any direction. A second entrance at ground level is open to almost stagnant air. How does this construction create an air flow through the burrow?

Pamela Zilly/The Image Bank

Figure Q14.28

29. An unopened can of diet cola floats when placed in a tank of water, whereas a can of regular cola of the same brand sinks in the tank. What do you suppose could explain this behavior?

30. Figure Q14.30 shows a glass cylinder containing four liquids of different densities. From top to bottom, the liquids are oil (orange), water (yellow), salt water (green), and mercury (silver). The cylinder also contains, from top to bottom, a Ping-Pong ball, a piece of wood, an egg, and a steel ball. (a) Which of these liquids has the lowest density, and which has the greatest? (b) What can you conclude about the density of each object?

Henry Leap and Jim Lehman

Figure Q14.30

31. In Figure Q14.31, an air stream moves from right to left through a tube that is constricted at the middle. Three Ping-Pong balls are levitated in equilibrium above the vertical columns through which the air escapes. (a) Why is the ball at the right higher than the one in the middle? (b) Why is the ball at the left lower than the ball at the right even though the horizontal tube has the same dimensions at these two points?

Henry Leap and Jim Lehman

Figure Q14.31

32. You are a passenger on a spacecraft. For your survival and comfort, the interior contains air just like that at the surface of the Earth. The craft is coasting through a very empty region of space. That is, a nearly perfect vacuum exists just outside the wall. Suddenly, a meteoroid pokes a hole, about the size of a large coin, right through the wall next to your seat. What will happen? Is there anything you can or should do about it?

PROBLEMS

1, 2, 3 = straightforward, intermediate, challenging ☐ = full solution available in the *Student Solutions Manual and Study Guide*

= coached solution with hints available at http://www.pse6.com = computer useful in solving problem

= paired numerical and symbolic problems

Section 14.1 Pressure

1. Calculate the mass of a solid iron sphere that has a diameter of 3.00 cm.

2. Find the order of magnitude of the density of the nucleus of an atom. What does this result suggest concerning the structure of matter? Model a nucleus as protons and neutrons closely packed together. Each has mass 1.67×10^{-27} kg and radius on the order of 10^{-15} m.

3. A 50.0-kg woman balances on one heel of a pair of high-heeled shoes. If the heel is circular and has a radius of 0.500 cm, what pressure does she exert on the floor?

4. The four tires of an automobile are inflated to a gauge pressure of 200 kPa. Each tire has an area of 0.024 0 m^2 in contact with the ground. Determine the weight of the automobile.

5. What is the total mass of the Earth's atmosphere? (The radius of the Earth is 6.37×10^6 m, and atmospheric pressure at the surface is 1.013×10^5 N/m^2.)

Section 14.2 Variation of Pressure with Depth

6. (a) Calculate the absolute pressure at an ocean depth of 1 000 m. Assume the density of seawater is 1 024 kg/m^3 and that the air above exerts a pressure of 101.3 kPa. (b) At this depth, what force must the frame around a circular submarine porthole having a diameter of 30.0 cm exert to counterbalance the force exerted by the water?

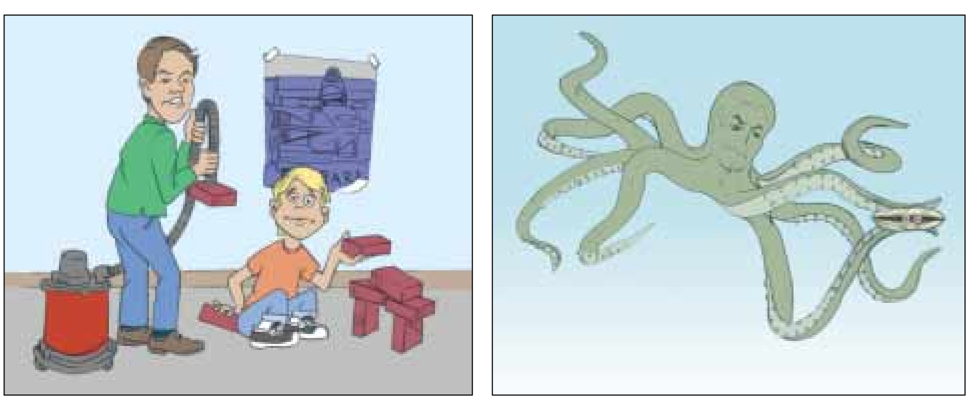
7. The spring of the pressure gauge shown in Figure 14.2 has a force constant of 1 000 N/m, and the piston has a diameter of 2.00 cm. As the gauge is lowered into water, what change in depth causes the piston to move in by 0.500 cm?

8. The small piston of a hydraulic lift has a cross-sectional area of 3.00 cm^2, and its large piston has a cross-sectional area of 200 cm^2 (Figure 14.4). What force must be applied to the small piston for the lift to raise a load of 15.0 kN? (In service stations, this force is usually exerted by compressed air.)

9. What must be the contact area between a suction cup (completely exhausted) and a ceiling if the cup is to support the weight of an 80.0-kg student?

10. (a) A very powerful vacuum cleaner has a hose 2.86 cm in diameter. With no nozzle on the hose, what is the weight of the heaviest brick that the cleaner can lift? (Fig. P14.10a) (b) **What If?** A very powerful octopus uses one sucker of diameter 2.86 cm on each of the two shells of a clam in an attempt to pull the shells apart (Fig. P14.10b). Find the greatest force the octopus can exert in salt water 32.3 m deep. *Caution*: Experimental verification can be interesting, but do not drop a brick on your foot. Do not overheat the motor of a vacuum cleaner. Do not get an octopus mad at you.

11. For the cellar of a new house, a hole is dug in the ground, with vertical sides going down 2.40 m. A concrete foundation wall is built all the way across the 9.60-m width of the



(a) (b)

Figure P14.10

excavation. This foundation wall is 0.183 m away from the front of the cellar hole. During a rainstorm, drainage from the street fills up the space in front of the concrete wall, but not the cellar behind the wall. The water does not soak into the clay soil. Find the force the water causes on the foundation wall. For comparison, the weight of the water is given by $2.40\text{ m} \times 9.60\text{ m} \times 0.183\text{ m} \times 1\,000\text{ kg/m}^3 \times 9.80\text{ m/s}^2 = 41.3\text{ kN}$.

12. A swimming pool has dimensions 30.0 m × 10.0 m and a flat bottom. When the pool is filled to a depth of 2.00 m with fresh water, what is the force caused by the water on the bottom? On each end? On each side?

13. A sealed spherical shell of diameter d is rigidly attached to a cart, which is moving horizontally with an acceleration a as in Figure P14.13. The sphere is nearly filled with a fluid having density ρ and also contains one small bubble of air at atmospheric pressure. Determine the pressure P at the center of the sphere.

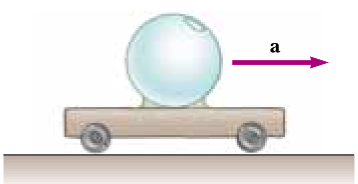


Figure P14.13

14. The tank in Figure P14.14 is filled with water 2.00 m deep. At the bottom of one side wall is a rectangular hatch 1.00 m high and 2.00 m wide, which is hinged at the top of the hatch. (a) Determine the force the water exerts on the hatch. (b) Find the torque exerted by the water about the hinges.

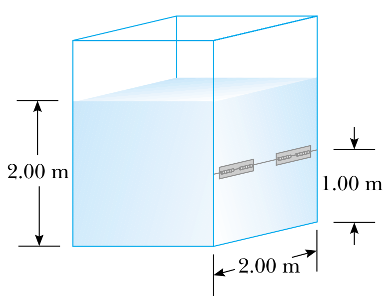


Figure P14.14

15. Review problem. The Abbott of Aberbrothock paid to have a bell moored to the Inchcape Rock to warn seamen of the hazard. Assume the bell was 3.00 m in diameter, cast from brass with a bulk modulus of $14.0 \times 10^{10}\text{ N/m}^2$. The pirate Ralph the Rover cut loose the warning bell and threw it into the ocean. By how much did the diameter of the bell decrease as it sank to a depth of 10.0 km? Years later, Ralph drowned when his ship collided with the rock. *Note*: The brass is compressed uniformly, so you may model the bell as a sphere of diameter 3.00 m.

Section 14.3 Pressure Measurements

16. Figure P14.16 shows Superman attempting to drink water through a very long straw. With his great strength he achieves maximum possible suction. The walls of the tubular straw do not collapse. (a) Find the maximum height through which he can lift the water. (b) **What If?** Still thirsty, the Man of Steel repeats his attempt on the Moon, which has no atmosphere. Find the difference between the water levels inside and outside the straw.

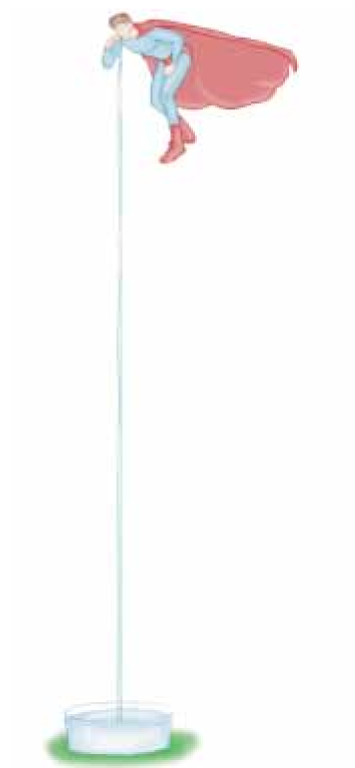


Figure P14.16

17. Blaise Pascal duplicated Torricelli's barometer using a red Bordeaux wine, of density 984 kg/m³, as the working liquid (Fig. P14.17). What was the height h of the wine

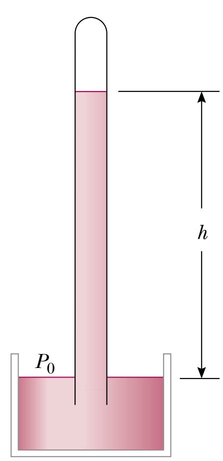


Figure P14.17

column for normal atmospheric pressure? Would you expect the vacuum above the column to be as good as for mercury?

18. Mercury is poured into a U-tube as in Figure P14.18a. The left arm of the tube has cross-sectional area A_1 of 10.0 cm^2, and the right arm has a cross-sectional area A_2 of 5.00 cm^2. One hundred grams of water are then poured into the right arm as in Figure P14.18b. (a) Determine the length of the water column in the right arm of the U-tube. (b) Given that the density of mercury is 13.6 g/cm^3, what distance h does the mercury rise in the left arm?

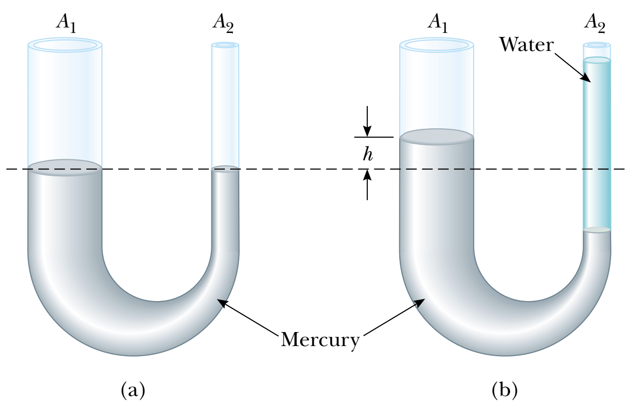


Figure P14.18

19. Normal atmospheric pressure is 1.013×10^5 Pa. The approach of a storm causes the height of a mercury barometer to drop by 20.0 mm from the normal height. What is the atmospheric pressure? (The density of mercury is 13.59 g/cm^3.)

20. A U-tube of uniform cross-sectional area, open to the atmosphere, is partially filled with mercury. Water is then poured into both arms. If the equilibrium configuration of the tube is as shown in Figure P14.20, with $h_2 = 1.00$ cm, determine the value of h_1.

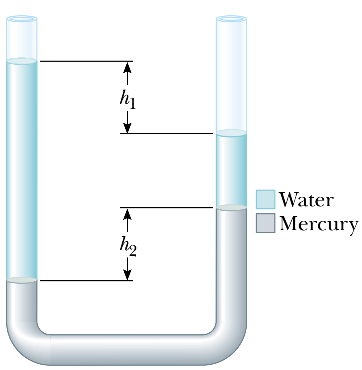


Figure P14.20

21. The human brain and spinal cord are immersed in the cerebrospinal fluid. The fluid is normally continuous between the cranial and spinal cavities. It normally exerts a pressure of 100 to 200 mm of H_2O above the prevailing atmospheric pressure. In medical work pressures are often measured in units of millimeters of H_2O because body fluids, including the cerebrospinal fluid, typically have the same density as water. The pressure of the cerebrospinal fluid can be measured by means of a *spinal tap*, as illustrated in Figure P14.21. A hollow tube is inserted into the spinal column, and the height to which the fluid rises is observed. If the fluid rises to a height of 160 mm, we write its gauge pressure as 160 mm H_2O. (a) Express this pressure in pascals, in atmospheres, and in millimeters of mercury. (b) Sometimes it is necessary to determine if an accident victim has suffered a crushed vertebra that is blocking flow of the cerebrospinal fluid in the spinal column. In other cases a physician may suspect a tumor or other growth is blocking the spinal column and inhibiting flow of cerebrospinal fluid. Such conditions can be investigated by means of the *Queckensted test*. In this procedure, the veins in the patient's neck are compressed, to make the blood pressure rise in the brain. The increase in pressure in the blood vessels is transmitted to the cerebrospinal fluid. What should be the normal effect on the height of the fluid in the spinal tap? (c) Suppose that compressing the veins had no effect on the fluid level. What might account for this?

Figure P14.21

Section 14.4 Buoyant Forces and Archimedes's Principle

22. (a) A light balloon is filled with 400 m^3 of helium. At 0°C, the balloon can lift a payload of what mass? (b) **What If?** In Table 14.1, observe that the density of hydrogen is nearly half the density of helium. What load can the balloon lift if filled with hydrogen?

$\boxed{\textbf{23.}}$ A Ping-Pong ball has a diameter of 3.80 cm and average density of 0.084 0 g/cm^3. What force is required to hold it completely submerged under water?

24. A Styrofoam slab has thickness h and density ρ_s. When a swimmer of mass m is resting on it, the slab floats in fresh water with its top at the same level as the water surface. Find the area of the slab.

25. A piece of aluminum with mass 1.00 kg and density 2 700 kg/m^3 is suspended from a string and then completely immersed in a container of water (Figure P14.25). Calculate the tension in the string (a) before and (b) after the metal is immersed.

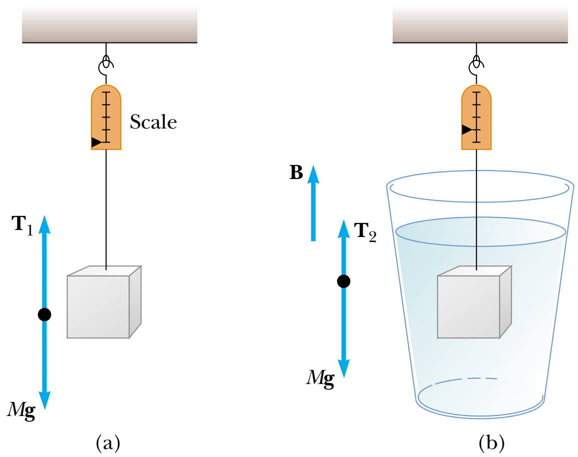


Figure P14.25 Problems 25 and 27

26. The weight of a rectangular block of low-density material is 15.0 N. With a thin string, the center of the horizontal bottom face of the block is tied to the bottom of a beaker partly filled with water. When 25.0% of the block's volume is submerged, the tension in the string is 10.0 N. (a) Sketch a free-body diagram for the block, showing all forces acting on it. (b) Find the buoyant force on the block. (c) Oil of density 800 kg/m^3 is now steadily added to the beaker, forming a layer above the water and surrounding the block. The oil exerts forces on each of the four side walls of the block that the oil touches. What are the directions of these forces? (d) What happens to the string tension as the oil is added? Explain how the oil has this effect on the string tension. (e) The string breaks when its tension reaches 60.0 N. At this moment, 25.0% of the block's volume is still below the water line; what additional fraction of the block's volume is below the top surface of the oil? (f) After the string breaks, the block comes to a new equilibrium position in the beaker. It is now in contact only with the oil. What fraction of the block's volume is submerged?

27. A 10.0-kg block of metal measuring 12.0 cm $\times$ 10.0 cm $\times$ 10.0 cm is suspended from a scale and immersed in water as in Figure P14.25b. The 12.0-cm dimension is vertical, and the top of the block is 5.00 cm below the surface of the water. (a) What are the forces acting on the top and on the bottom of the block? (Take $P_0 = 1.013\,0 \times 10^5$ N/m^2.) (b) What is the reading of the spring scale? (c) Show that the buoyant force equals the difference between the forces at the top and bottom of the block.

28. To an order of magnitude, how many helium-filled toy balloons would be required to lift you? Because helium is an irreplaceable resource, develop a theoretical answer rather than an experimental answer. In your solution state what physical quantities you take as data and the values you measure or estimate for them.

29. A cube of wood having an edge dimension of 20.0 cm and a density of 650 kg/m^3 floats on water. (a) What is the distance from the horizontal top surface of the cube to the water level? (b) How much lead weight has to be placed on top of the cube so that its top is just level with the water?

30. A spherical aluminum ball of mass 1.26 kg contains an empty spherical cavity that is concentric with the ball. The ball just barely floats in water. Calculate (a) the outer radius of the ball and (b) the radius of the cavity.

31. Determination of the density of a fluid has many important applications. A car battery contains sulfuric acid, for which density is a measure of concentration. For the battery to function properly, the density must be inside a range specified by the manufacturer. Similarly, the effectiveness of antifreeze in your car's engine coolant depends on the density of the mixture (usually ethylene glycol and water). When you donate blood to a blood bank, its screening includes determination of the density of the blood, since higher density correlates with higher hemoglobin content. A *hydrometer* is an instrument used to determine liquid density. A simple one is sketched in Figure P14.31. The bulb of a syringe is squeezed and released to let the atmosphere lift a sample of the liquid of interest into a tube containing a calibrated rod of known density. The rod, of length L and average density ρ_0, floats partially immersed in the liquid of density ρ. A length h of the rod protrudes above the surface of the liquid. Show that the density of the liquid is given by

$$\rho = \frac{\rho_0 L}{L - h}$$

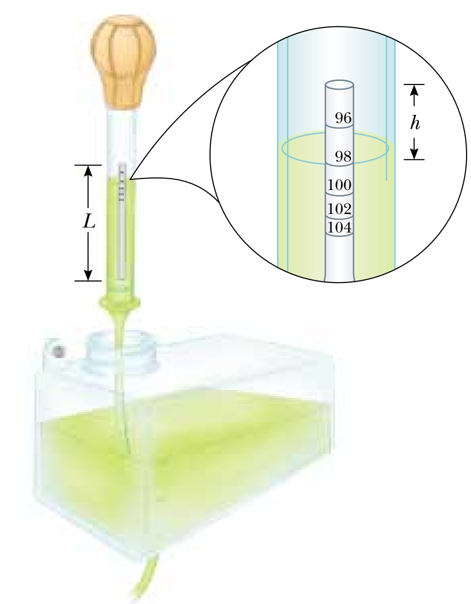


Figure P14.31 Problems 31 and 32

32. Refer to Problem 31 and Figure P14.31. A hydrometer is to be constructed with a cylindrical floating rod. Nine fiduciary marks are to be placed along the rod to indicate densities of 0.98 g/cm^3, 1.00 g/cm^3, 1.02 g/cm^3, 1.04 g/cm^3, . . . 1.14 g/cm^3. The row of marks is to start 0.200 cm from the top end of the rod and end 1.80 cm from the top end. (a) What is the required length of the rod? (b) What must be its average density? (c) Should the marks be equally spaced? Explain your answer.

33. How many cubic meters of helium are required to lift a balloon with a 400-kg payload to a height of 8 000 m? (Take $\rho_{He} = 0.180$ kg/m^3.) Assume that the balloon maintains a constant volume and that the density of air decreases with the altitude z according to the expression $\rho_{air} = \rho_0 e^{-z/8\,000}$, where z is in meters and $\rho_0 = 1.25\ kg/m^3$ is the density of air at sea level.

34. A frog in a hemispherical pod (Fig. P14.34) just floats without sinking into a sea of blue-green ooze with density $1.35\ g/cm^3$. If the pod has radius 6.00 cm and negligible mass, what is the mass of the frog?

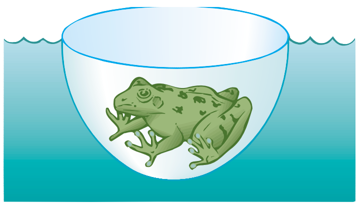


Figure P14.34

35. A plastic sphere floats in water with 50.0 percent of its volume submerged. This same sphere floats in glycerin with 40.0 percent of its volume submerged. Determine the densities of the glycerin and the sphere.

36. A bathysphere used for deep-sea exploration has a radius of 1.50 m and a mass of 1.20×10^4 kg. To dive, this submarine takes on mass in the form of seawater. Determine the amount of mass the submarine must take on if it is to descend at a constant speed of 1.20 m/s, when the resistive force on it is 1 100 N in the upward direction. The density of seawater is $1.03 \times 10^3\ kg/m^3$.

37. The United States possesses the eight largest warships in the world—aircraft carriers of the *Nimitz* class—and is building two more. Suppose one of the ships bobs up to float 11.0 cm higher in the water when 50 fighters take off from it in 25 min, at a location where the free-fall acceleration is $9.78\ m/s^2$. Bristling with bombs and missiles, the planes have average mass 29 000 kg. Find the horizontal area enclosed by the waterline of the $4-billion ship. By comparison, its flight deck has area 18 000 m^2. Below decks are passageways hundreds of meters long, so narrow that two large men cannot pass each other.

Section 14.5 Fluid Dynamics
Section 14.6 Bernoulli's Equation

38. A horizontal pipe 10.0 cm in diameter has a smooth reduction to a pipe 5.00 cm in diameter. If the pressure of the water in the larger pipe is 8.00×10^4 Pa and the pressure in the smaller pipe is 6.00×10^4 Pa, at what rate does water flow through the pipes?

39. A large storage tank, open at the top and filled with water, develops a small hole in its side at a point 16.0 m below the water level. If the rate of flow from the leak is equal to $2.50 \times 10^{-3}\ m^3/min$, determine (a) the speed at which the water leaves the hole and (b) the diameter of the hole.

40. A village maintains a large tank with an open top, containing water for emergencies. The water can drain from the tank through a hose of diameter 6.60 cm. The hose ends with a nozzle of diameter 2.20 cm. A rubber stopper is inserted into the nozzle. The water level in the tank is kept 7.50 m above the nozzle. (a) Calculate the friction force exerted by the nozzle on the stopper. (b) The stopper is removed. What mass of water flows from the nozzle in 2.00 h? (c) Calculate the gauge pressure of the flowing water in the hose just behind the nozzle.

41. Water flows through a fire hose of diameter 6.35 cm at a rate of $0.0120\ m^3/s$. The fire hose ends in a nozzle of inner diameter 2.20 cm. What is the speed with which the water exits the nozzle?

42. Water falls over a dam of height h with a mass flow rate of R, in units of kg/s. (a) Show that the power available from the water is

$$\mathcal{P} = Rgh$$

where g is the free-fall acceleration. (b) Each hydroelectric unit at the Grand Coulee Dam takes in water at a rate of 8.50×10^5 kg/s from a height of 87.0 m. The power developed by the falling water is converted to electric power with an efficiency of 85.0%. How much electric power is produced by each hydroelectric unit?

43. Figure P14.43 shows a stream of water in steady flow from a kitchen faucet. At the faucet the diameter of the stream is 0.960 cm. The stream fills a 125-cm^3 container in 16.3 s. Find the diameter of the stream 13.0 cm below the opening of the faucet.

George Semple

Figure P14.43

44. A legendary Dutch boy saved Holland by plugging a hole in a dike with his finger, which is 1.20 cm in diameter. If the hole was 2.00 m below the surface of the North Sea (density 1 030 kg/m^3), (a) what was the force on his finger? (b) If he pulled his finger out of the hole, how long would it take the released water to fill 1 acre of land to a depth of 1 ft, assuming the hole remained constant in size? (A typical U.S. family of four uses 1 acre-foot of water, 1 234 m^3, in 1 year.)

45. Through a pipe 15.0 cm in diameter, water is pumped from the Colorado River up to Grand Canyon Village, located on the rim of the canyon. The river is at an elevation of 564 m, and the village is at an elevation of 2 096 m.

(a) What is the minimum pressure at which the water must be pumped if it is to arrive at the village? (b) If 4 500 m^3 are pumped per day, what is the speed of the water in the pipe? (c) What additional pressure is necessary to deliver this flow? *Note*: Assume that the free-fall acceleration and the density of air are constant over this range of elevations.

46. Old Faithful Geyser in Yellowstone Park (Fig. P14.46) erupts at approximately 1-h intervals, and the height of the water column reaches 40.0 m. (a) Model the rising stream as a series of separate drops. Analyze the free-fall motion of one of the drops to determine the speed at which the water leaves the ground. (b) **What If?** Model the rising stream as an ideal fluid in streamline flow. Use Bernoulli's equation to determine the speed of the water as it leaves ground level. (c) What is the pressure (above atmospheric) in the heated underground chamber if its depth is 175 m? You may assume that the chamber is large compared with the geyser's vent.

Stan Osolinski/Dembinsky Photo Associates

Figure P14.46

47. A Venturi tube may be used as a fluid flow meter (see Fig. 14.20). If the difference in pressure is $P_1 - P_2 = 21.0$ kPa, find the fluid flow rate in cubic meters per second, given that the radius of the outlet tube is 1.00 cm, the radius of the inlet tube is 2.00 cm, and the fluid is gasoline ($\rho = 700$ kg/m^3).

Section 14.7 Other Applications of Fluid Dynamics

48. An airplane has a mass of 1.60×10^4 kg, and each wing has an area of 40.0 m^2. During level flight, the pressure on the lower wing surface is 7.00×10^4 Pa. Determine the pressure on the upper wing surface.

49. A Pitot tube can be used to determine the velocity of air flow by measuring the difference between the total pressure and the static pressure (Fig. P14.49). If the fluid in the tube is mercury, density $\rho_{Hg} = 13\ 600$ kg/m^3, and $\Delta h = 5.00$ cm, find the speed of air flow. (Assume that the air is stagnant at point A, and take $\rho_{air} = 1.25$ kg/m^3.)

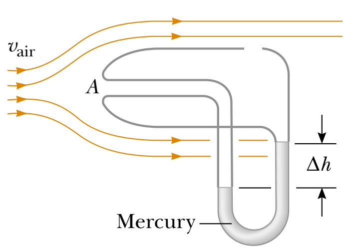


Figure P14.49

50. An airplane is cruising at an altitude of 10 km. The pressure outside the craft is 0.287 atm; within the passenger compartment the pressure is 1.00 atm and the temperature is 20°C. A small leak occurs in one of the window seals in the passenger compartment. Model the air as an ideal fluid to find the speed of the stream of air flowing through the leak.

51. A siphon is used to drain water from a tank, as illustrated in Figure P14.51. The siphon has a uniform diameter. Assume steady flow without friction. (a) If the distance $h = 1.00$ m, find the speed of outflow at the end of the siphon. (b) **What If?** What is the limitation on the height of the top of the siphon above the water surface? (For the flow of the liquid to be continuous, the pressure must not drop below the vapor pressure of the liquid.)

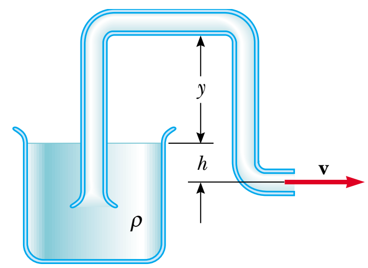


Figure P14.51

52. The Bernoulli effect can have important consequences for the design of buildings. For example, wind can blow around a skyscraper at remarkably high speed, creating low pressure. The higher atmospheric pressure in the still air inside the buildings can cause windows to pop out. As originally constructed, the John Hancock building in Boston popped window panes, which fell many stories to the sidewalk below. (a) Suppose that a horizontal wind blows in streamline flow with a speed of 11.2 m/s outside a large pane of plate glass with dimensions 4.00 m $\times$ 1.50 m. Assume the density of the air to be uniform at 1.30 kg/m^3. The air inside the building is at atmospheric pressure. What is the total force exerted by air on the window pane? (b) **What If?** If a second skyscraper is built nearby, the air speed can be especially high where wind passes through the narrow separation between the buildings. Solve part (a) again if the wind speed is 22.4 m/s, twice as high.

53. A hypodermic syringe contains a medicine with the density of water (Figure P14.53). The barrel of the syringe has a cross-sectional area $A = 2.50 \times 10^{-5}$ m^2, and the needle has a cross-sectional area $a = 1.00 \times 10^{-8}$ m^2. In the absence of a force on the plunger, the pressure everywhere is 1 atm. A force **F** of magnitude 2.00 N acts on the plunger, making medicine squirt horizontally from the needle. Determine the speed of the medicine as it leaves the needle's tip.

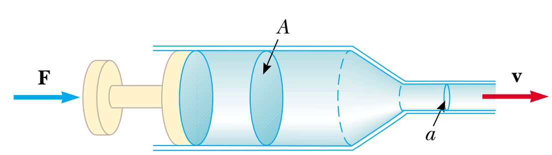


Figure P14.53

Additional Problems

54. Figure P14.54 shows a water tank with a valve at the bottom. If this valve is opened, what is the maximum height attained by the water stream coming out of the right side of the tank? Assume that $h = 10.0$ m, $L = 2.00$ m, and $\theta = 30.0°$, and that the cross-sectional area at A is very large compared with that at B.

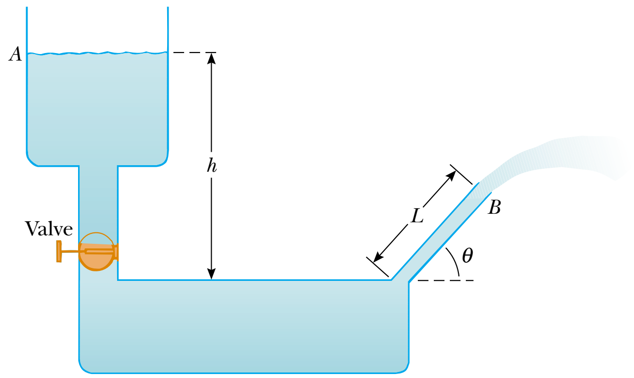


Figure P14.54

55. A helium-filled balloon is tied to a 2.00-m-long, 0.050 0-kg uniform string. The balloon is spherical with a radius of 0.400 m. When released, it lifts a length h of string and then remains in equilibrium, as in Figure P14.55. Determine the value of h. The envelope of the balloon has mass 0.250 kg.

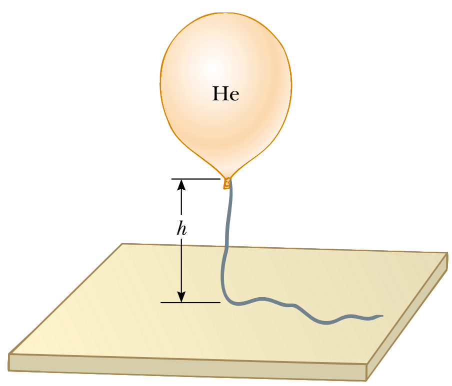


Figure P14.55

56. Water is forced out of a fire extinguisher by air pressure, as shown in Figure P14.56. How much gauge air pressure in the tank (above atmospheric) is required for the water jet to have a speed of 30.0 m/s when the water level in the tank is 0.500 m below the nozzle?

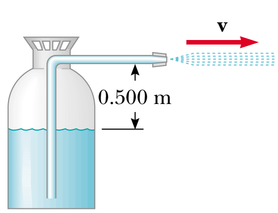


Figure P14.56

57. The true weight of an object can be measured in a vacuum, where buoyant forces are absent. An object of volume V is weighed in air on a balance with the use of weights of density ρ. If the density of air is ρ_{air} and the balance reads F_g', show that the true weight F_g is

$$F_g = F_g' + \left(V - \frac{F_g'}{\rho g}\right)\rho_{air}\, g$$

58. A wooden dowel has a diameter of 1.20 cm. It floats in water with 0.400 cm of its diameter above water (Fig. P14.58). Determine the density of the dowel.

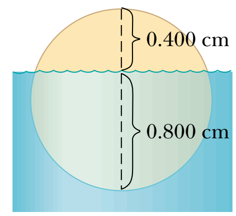


Figure P14.58

59. A light spring of constant $k = 90.0$ N/m is attached vertically to a table (Fig. P14.59a). A 2.00-g balloon is filled with helium (density = 0.180 kg/m^3) to a volume of 5.00 m^3 and is then connected to the spring, causing it to stretch as in Figure P14.59b. Determine the extension distance L when the balloon is in equilibrium.

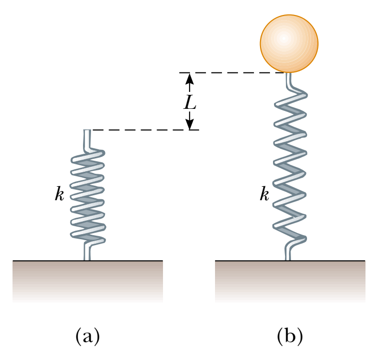


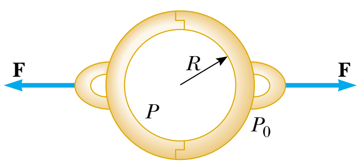
Figure P14.59

60. Evangelista Torricelli was the first person to realize that we live at the bottom of an ocean of air. He correctly surmised that the pressure of our atmosphere is attributable to the weight of the air. The density of air at 0°C at the Earth's surface is 1.29 kg/m^3. The density decreases with increasing altitude (as the atmosphere thins). On the other hand, if we assume that the density is constant at 1.29 kg/m^3 up to some altitude h, and zero above that altitude, then h would represent the depth of the ocean of air. Use this model to determine the value of h that gives a pressure of 1.00 atm at the surface of the Earth. Would the peak of

Mount Everest rise above the surface of such an atmosphere?

61. **Review problem**. With reference to Figure 14.5, show that the total torque exerted by the water behind the dam about a horizontal axis through O is $\frac{1}{6}\rho g w H^3$. Show that the effective line of action of the total force exerted by the water is at a distance $\frac{1}{3}H$ above O.

62. In about 1657 Otto von Guericke, inventor of the air pump, evacuated a sphere made of two brass hemispheres. Two teams of eight horses each could pull the hemispheres apart only on some trials, and then "with greatest difficulty," with the resulting sound likened to a cannon firing (Fig. P14.62). (a) Show that the force F required to pull the evacuated hemispheres apart is $\pi R^2(P_0 - P)$, where R is the radius of the hemispheres and P is the pressure inside the hemispheres, which is much less than P_0. (b) Determine the force if $P = 0.100P_0$ and $R = 0.300$ m.



The Granger Collection

Figure P14.62 The colored engraving, dated 1672, illustrates Otto von Guericke's demonstration of the force due to air pressure as performed before Emperor Ferdinand III in 1657.

63. A 1.00-kg beaker containing 2.00 kg of oil (density = 916.0 kg/m^3) rests on a scale. A 2.00-kg block of iron is suspended from a spring scale and completely submerged in the oil as in Figure P14.63. Determine the equilibrium readings of both scales.

64. A beaker of mass m_{beaker} containing oil of mass m_{oil} (density = ρ_{oil}) rests on a scale. A block of iron of mass m_{iron} is suspended from a spring scale and completely submerged in the oil as in Figure P14.63. Determine the equilibrium readings of both scales.

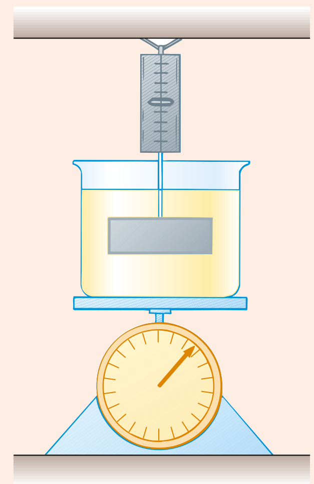


Figure P14.63 Problems 63 and 64

65. In 1983, the United States began coining the cent piece out of copper-clad zinc rather than pure copper. The mass of the old copper penny is 3.083 g, while that of the new cent is 2.517 g. Calculate the percentage of zinc (by volume) in the new cent. The density of copper is 8.960 g/cm^3 and that of zinc is 7.133 g/cm^3. The new and old coins have the same volume.

66. A thin spherical shell of mass 4.00 kg and diameter 0.200 m is filled with helium (density = 0.180 kg/m^3). It is then released from rest on the bottom of a pool of water that is 4.00 m deep. (a) Neglecting frictional effects, show that the shell rises with constant acceleration and determine the value of that acceleration. (b) How long will it take for the top of the shell to reach the water surface?

67. **Review problem.** A uniform disk of mass 10.0 kg and radius 0.250 m spins at 300 rev/min on a low-friction axle. It must be brought to a stop in 1.00 min by a brake pad that makes contact with the disk at average distance 0.220 m from the axis. The coefficient of friction between pad and disk is 0.500. A piston in a cylinder of diameter 5.00 cm presses the brake pad against the disk. Find the pressure required for the brake fluid in the cylinder.

68. Show that the variation of atmospheric pressure with altitude is given by $P = P_0 e^{-\alpha y}$, where $\alpha = \rho_0 g/P_0$, P_0 is atmospheric pressure at some reference level $y = 0$, and ρ_0 is the atmospheric density at this level. Assume that the decrease in atmospheric pressure over an infinitesimal change in altitude (so that the density is approximately uniform) is given by $dP = -\rho g\, dy$, and that the density of air is proportional to the pressure.

69. An incompressible, nonviscous fluid is initially at rest in the vertical portion of the pipe shown in Figure P14.69a, where $L = 2.00$ m. When the valve is opened, the fluid flows into the horizontal section of the pipe. What is the speed of the fluid when all of it is in the horizontal section, as in Figure P14.69b? Assume the cross-sectional area of the entire pipe is constant.

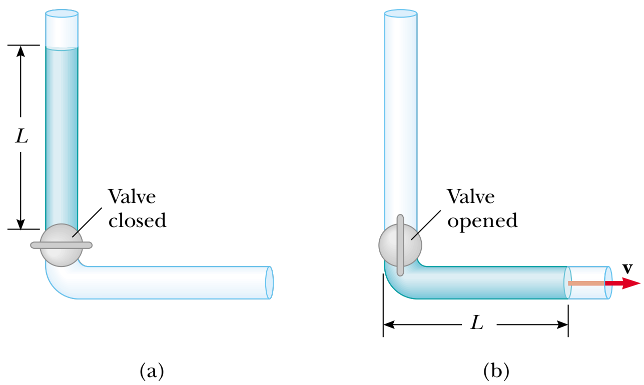


Figure P14.69

70. A cube of ice whose edges measure 20.0 mm is floating in a glass of ice-cold water with one of its faces parallel to the water's surface. (a) How far below the water surface is the bottom face of the block? (b) Ice-cold ethyl alcohol is gently poured onto the water surface to form a layer 5.00 mm thick above the water. The alcohol does not mix with the water. When the ice cube again attains hydrostatic equilibrium, what will be the distance from the top of the water to the bottom face of the block? (c) Additional cold ethyl alcohol is poured onto the water's surface until the top surface of the alcohol coincides with the top surface of the ice cube (in hydrostatic equilibrium). How thick is the required layer of ethyl alcohol?

71. A U-tube open at both ends is partially filled with water (Fig. P14.71a). Oil having a density of 750 kg/m^3 is then poured into the right arm and forms a column $L = 5.00$ cm high (Fig. P14.71b). (a) Determine the difference h in the heights of the two liquid surfaces. (b) The right arm is then shielded from any air motion while air is blown across the top of the left arm until the surfaces of the two liquids are at the same height (Fig. P14.71c). Determine the speed of the air being blown across the left arm. Take the density of air as 1.29 kg/m^3.

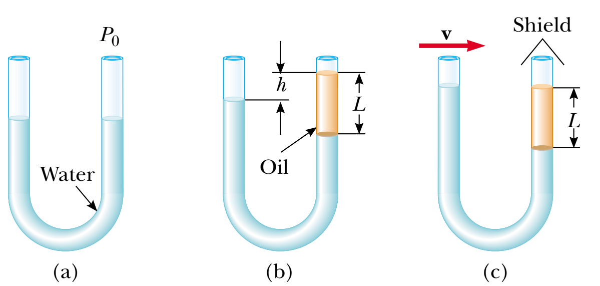


Figure P14.71

72. The water supply of a building is fed through a main pipe 6.00 cm in diameter. A 2.00-cm-diameter faucet tap, located 2.00 m above the main pipe, is observed to fill a 25.0-L container in 30.0 s. (a) What is the speed at which the water leaves the faucet? (b) What is the gauge pressure in the 6-cm main pipe? (Assume the faucet is the only "leak" in the building.)

73. The *spirit-in-glass thermometer*, invented in Florence, Italy, around 1654, consists of a tube of liquid (the spirit) containing a number of submerged glass spheres with slightly different masses (Fig. P14.73). At sufficiently low temperatures all the spheres float, but as the temperature rises, the spheres sink one after another. The device is a crude but interesting tool for measuring temperature. Suppose that the tube is filled with ethyl alcohol, whose density is 0.789 45 g/cm^3 at 20.0°C and decreases to 0.780 97 g/cm^3 at 30.0°C. (a) If one of the spheres has a radius of 1.000 cm and is in equilibrium halfway up the tube at 20.0°C, determine its mass. (b) When the temperature increases to 30.0°C, what mass must a second sphere of the same radius have in order to be in equilibrium at the halfway point? (c) At 30.0°C the first sphere has fallen to the bottom of the tube. What upward force does the bottom of the tube exert on this sphere?

Courtesy of Jeanne Maier

Figure P14.73

74. A woman is draining her fish tank by siphoning the water into an outdoor drain, as shown in Figure P14.74. The rectangular tank has footprint area A and depth h. The drain is located a distance d below the surface of the water in the tank, where $d \gg h$. The cross-sectional area of the siphon tube is A'. Model the water as flowing without friction. (a) Show that the time interval required to empty the tank is given by

$$\Delta t = \frac{Ah}{A'\sqrt{2gd}}$$

(b) Evaluate the time interval required to empty the tank if it is a cube 0.500 m on each edge, if $A' = 2.00\ cm^2$, and $d = 10.0$ m.

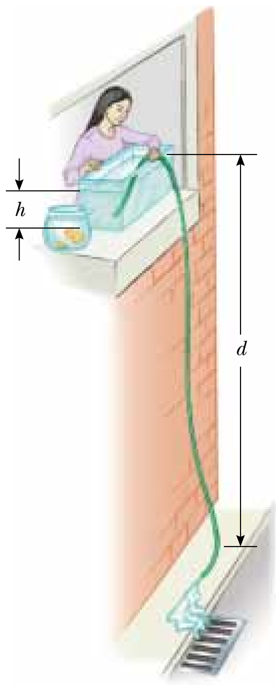


Figure P14.74

75. The hull of an experimental boat is to be lifted above the water by a hydrofoil mounted below its keel, as shown in Figure P14.75. The hydrofoil has a shape like that of an airplane wing. Its area projected onto a horizontal surface is A. When the boat is towed at sufficiently high speed, water of density ρ moves in streamline flow so that its average speed at the top of the hydrofoil is n times larger than its speed v_b below the hydrofoil. (a) Neglecting the buoyant force, show that the upward lift force exerted by the water on the hydrofoil has a magnitude given by

$$F \approx \tfrac{1}{2}(n^2 - 1)\rho {v_b}^2 A$$

(b) The boat has mass M. Show that the liftoff speed is given by

$$v \approx \sqrt{\frac{2Mg}{(n^2 - 1)A\rho}}$$

(c) Assume that an 800-kg boat is to lift off at 9.50 m/s. Evaluate the area A required for the hydrofoil if its design yields $n = 1.05$.

Figure P14.75

Answer to Quick Quizzes

14.1 (a). Because the basketball player's weight is distributed over the larger surface area of the shoe, the pressure (F/A) that he applies is relatively small. The woman's lesser weight is distributed over the very small cross-sectional area of the spiked heel, so the pressure is high.

14.2 (a). Because both fluids have the same depth, the one with the smaller density (alcohol) will exert the smaller pressure.

14.3 (c). All barometers will have the same pressure at the bottom of the column of fluid—atmospheric pressure. Thus, the barometer with the highest column will be the one with the fluid of lowest density.

14.4 (d). Because there is no atmosphere on the Moon, there is no atmospheric pressure to provide a force to push the water up the straw.

14.5 (b). For a totally submerged object, the buoyant force does not depend on the depth in an incompressible fluid.

14.6 (c). The ice cube displaces a volume of water that has a weight equal to that of the ice cube. When the ice cube melts, it becomes a parcel of water with the same weight and exactly the volume that was displaced by the ice cube before.

14.7 (b) or (c). In all three cases, the weight of the treasure chest causes a downward force on the raft that makes it sink into the water. In (b) and (c), however, the treasure chest also displaces water, which provides a buoyant force in the upward direction, reducing the effect of the chest's weight.

14.8 (b). The liquid moves at the highest speed in the straw with the smaller cross sectional area.

14.9 (a). The high-speed air between the balloons results in low pressure in this region. The higher pressure on the outer surfaces of the balloons pushes them toward each other.